Extra problems I

(1) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(2) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \cos 2x + \cos 5x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(3) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(4) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. Ininially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(5) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = \sin 3x$ for $0 < x < \pi$

(6) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = \sin 2y$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for

(7) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Find the Fourier Series for $f(x)$.

(8) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(9) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(10) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = x$ for $0 \le x \le \pi/2$, $u(x,0) = \pi/2$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at $\pi/2$. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(11) A function $f(x)$ is defined by: $f(x) = x$ for $0 \le x < \pi/2$ and $f(x) = \pi - x$ for $\pi/2 \le x < \pi$. Extend $f(x)$ as an odd function for $-\pi \le x < \pi$. Extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \le x \le 4\pi$.

Calculate the Fourier Series for $f(x)$.

(12) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$, and the temperature at each end is fixed at 0. The initial temperature is $u(x,0) = \begin{cases} 1, & \text{for } 0 \le x < \frac{\pi}{2} \\ -1, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$ Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(13) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ 1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π.

(13a) Sketch the graph of f for $-2\pi \le x \le 4\pi$.

(13b) Find the Fourier Series for $f(x)$.

(14) A function $f(x)$ is defined by: $f(x) = x$ for $-\pi \le x < \pi$ and $f(x)$ is extended periodically with period 2π. Find the Fourier Series for $f(x)$.

(15) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = \cos x + \cos 3x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(16) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin 3x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(17) A string length π satisfies the wave equation $u_{tt} = 9u_{xx}$. Initially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u_t(x,0) = \sin 2x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(18) Find the solution of Laplace's Equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for $0 < x < \pi$

(19) Find the solution of Laplace's Equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = \sin 2x$, $0 < x < \pi$; and $u(x,\pi) = 0$ for $0 < x < \pi$

(20) $f(x) = \begin{cases} x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended as an <u>even</u> function for $-\pi \le x < \pi$, and then $f(x)$ is extended periodically with period 2π. Find the Fourier Series for $f(x)$.

Extra Problems II

(21) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(22) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \cos 2x + \cos 5x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(23) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(24) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. Ininially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(25) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = \sin 3x$ for $0 < x < \pi$

(26) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = \sin 2y$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for

(27) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Find the Fourier Series for $f(x)$.

(28) $f(x) = x^2$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>odd</u> function of period 2π. Find the Fourier Series for $f(x)$.

(29) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(30) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

EXTRA PROBLEMS III

(31) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = x$ for $0 \le x \le \pi/2$, $u(x,0) = \pi/2$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at $\pi/2$. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(32) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(33) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \cos 2x + \cos 5x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(34) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(35) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. Ininially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(36) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = \sin 3x$ for $0 < x < \pi$

(37) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = \sin 2y$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for

(38) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Find the Fourier Series for $f(x)$.

(39) $f(x) = x^2$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>odd</u> function of period 2π. Find the Fourier Series for $f(x)$.

(40) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

EXTRA PROBLEMS IV

(41) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(42) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = x$ for $0 \le x \le \pi/2$, $u(x,0) = \pi/2$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at $\pi/2$. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(43) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ 1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π.

(43a) Sketch the graph of f for $-2\pi \le x \le 4\pi$.

(43b) Find the Fourier Series for $f(x)$.

(44) The temperature in a rod of length 1 satisfies the heat equation $100u_t = u_{xx}$. The temperature initially is $u(x,0) = \sin 2\pi x + \sin 5\pi x$ for $0 \le x \le 1$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(45) A function $f(x)$ is defined by: $f(x) = x$ for $-\pi \le x < \pi$ and $f(x)$ is extended periodically with period 2π. Find the Fourier Series for $f(x)$.

(46) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = \cos x + \cos 3x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(47) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin 3x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(48) A string length π satisfies the wave equation $u_{tt} = 9u_{xx}$. Initially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u_t(x,0) = \sin 2x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(49) Find the solution of Laplace's Equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for $0 < x < \pi$

(50) Find the solution of Laplace's Equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 \le x \le \pi$,

$0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = \sin 2x$, $0 < x < \pi$; and $u(x,\pi) = 0$ for $0 < x < \pi$

EXTRA PROBLEMS V

(51) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(52) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \cos 2x + \cos 5x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(53) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(54) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. Ininially $u(x,0) = 0$ for $0 \le x \le \pi$. The initial velocity is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(55) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi,y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = \sin 3x$ for $0 < x < \pi$

(56) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = \sin 2y$, for $0 < y < \pi$; $u(\pi,y) = 0$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = 0$ for

(57) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Find the Fourier Series for $f(x)$.

(58) $f(x) = x^2$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>odd</u> function of period 2π. Find the Fourier Series for $f(x)$.

(59) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(60) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; Find the Fourier Series for $f(x)$.

(71) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = x$ for $0 \le x \le \pi/2$, $u(x,0) = \pi/2$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at $\pi/2$. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.