M309 Sample Final #1 Dec 6, 2000 Name: ....

(1a) Find the general solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$

(1b) Classify the origin of the system in (3a) as node, saddle, center or spiral.

(1c) Is system in (3a) as stable or unstable.

(1d) Find the solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$ with with $x_1(0) = 2$ and $x_2(0) = -4$. 1

(2) The populations of two species satisfy: $x' = x(1 - x + 0.5y)$; $y' = y(2.5 - 1.5y + 0.25x)$.

(2a) Find all critical points of the system.

(2b) For each critical point, find the corresponding approximate linear system.

(2c) For each critical point, find the eigenvalues, eigenvectors of the approximate linear system.

(2d) Classify each critical point as to type and stability.

(2e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.

(4) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(5) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \cos 2x + \cos 5x$ for $0 \le x \le \pi$, and the ends are insulated. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(6) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = x$ for $0 \le x \le \pi/2$, $u(x,0) = \pi/2$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at $\pi/2$. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(7) A string length π satisfies the wave equation $u_{tt} = 16u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$. The initial velocity is $u(x,0) = \sin 2x + \sin 3x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(8) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0,y) = 0$, for $0 < y < \pi$; $u(\pi, y) = \sin 3y$ for $0 < y < \pi$; $u(x,0) = 0$, $0 < x < \pi$; and $u(x,\pi) = \sin 3x$ for $0 < x < \pi$

(9) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π.

(9a) Find the Fourier Series for $f(x)$.

(9b) Write Parseval's equality for $f(x)$.

(10) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$

(10a) Calculate the Fourier Series for $f(x)$.

(1b) Write Parseval's equality for $f(x)$.