M309 Sample Final #2 Name: ....

(1a) Let $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$. Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

(1c) Describe the behavior of $x(t)$ as $t \longrightarrow \infty$. Sketch the trajectory.

(2) The populations of two species satisfy: $x' = x(1 - 0.5x - 0.5y)$; $y' = y(-0.25 + 0.5x)$.

(2a) Find all critical points of the system.

(2b) For each critical point, find the corresponding approximate linear system.

(2c) For each critical point, find the eigenvalues, eigenvectors of the approximate linear system.

(2d) Classify each critical point as to type and stability.

(2e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.

(3) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ |x| + 1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π.
Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$.

(4) A function $f(x)$ is defined by: $f(x) = x$ for $0 \le x < \pi/2$ and $f(x) = \pi - x$ for $\pi/2 \le x < \pi$. Extend $f(x)$ as an even function for $-\pi \le x < \pi$. Extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \le x \le 4\pi$.
Calculate the Fourier Series for $f(x)$.

(5) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$, and the temperature at each end is fixed at 0. The initial temperature is $u(x,0) = \begin{cases} 1, & \text{for } 0 \le x < \frac{\pi}{2} \\ -1, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$
Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(6) $f(x) = \begin{cases} x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended as an <u>even</u> function for $-\pi \le x < \pi$, and then $f(x)$ is extended periodically with period 2π. Find the Fourier Series for $f(x)$.

(7) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>odd</u> function of period 2π. Find the Fourier Series for $f(x)$.

(8) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ 1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$

(8a) Find the Fourier Series for $f(x)$.

(8b) Write Parseval's equality for $f(x)$.

(9) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < 0 \\ x, & \text{for } 0 \le x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$

(9a) Find the Fourier Series for $f(x)$.

(9b) Write Parseval's equality for $f(x)$.

(10) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$. The temperature initially is $u(x,0) = 0$ for $0 \le x \le \pi/2$, $u(x,0) = 1$ for $\pi/2 \le x \le \pi$. The temperature at $x = 0$ is fixed at 0. The temperature at $x = \pi$ is fixed at 1. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.