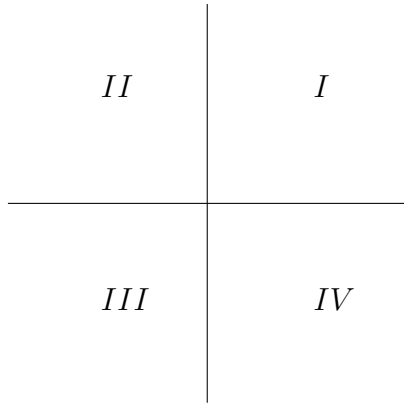


(1a) Let $A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

(1c) The quadrants are numbered in the diagram below The trajectory of (1b) starts in quadrant *I*. In which quadrant (*I*, *II*, *III* or *IV*) will the trajectory be when $t = 1000$?



(2a) Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(2b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

(2c) The four quadrants are numbered in the diagram of (1c). The trajectory of (1b) starts in quadrant *I*. What are the next four quadrants through which the trajectory passes?

(3a) Let $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(3b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$.

(4a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

(4b) Find a matrix T and a diagonal matrix D so that $T^{-1}AT = D$.

(5a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & 1 \\ -3 & 2 \end{bmatrix}$.

(5b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(6a) Find the general solution to
$$\begin{aligned} x_1' &= 6x_1 + x_2 \\ x_2' &= -3x_1 + 2x_2 \end{aligned}$$

(6b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

(7a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 7 & 5 \\ -1 & 1 \end{bmatrix}$.

(7b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(8a) Find the general solution to
$$\begin{aligned} x_1' &= 7x_1 + 5x_2 \\ x_2' &= -x_1 + x_2 \end{aligned}$$

(8b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

(9) Find the general solution to the system
$$\begin{aligned} x_1' &= x_1 + x_2 + e^t \\ x_2' &= 4x_1 + x_2 + -2e^t \end{aligned}$$

(10) (Section 9.2 # 11:) $(0.5, 0.5)$ is a critical point of the system
$$\begin{cases} x' = -x + 2xy \\ y' = y - x^2 - y^2 \end{cases}$$

For the critical point $(0.5, 0.5)$ find the corresponding approximate linear system and find the eigenvalues and eigenvectors. Classify the critical point $(0.5, 0.5)$ by type, and determine whether or not it is **AS** (asymptotically stable), **S** but not **AS** (stable but not asymptotically stable), or **US** (unstable).

(11a) Find the general solution to the system
$$\begin{aligned}x_1' &= x_1 + x_2 \\x_2' &= 4x_1 + x_2\end{aligned}$$

(11b) Find the solution to the system
$$\begin{aligned}x_1' &= x_1 + x_2 \\x_2' &= 4x_1 + x_2\end{aligned}$$
 with $x_1(1) = 3, x_2(1) = 2$.

(12) Find all critical points of the system:
$$\begin{cases}x' = x(1.5 - 0.5x - y) \\y' = y(0.75 - y - 0.125x)\end{cases}$$

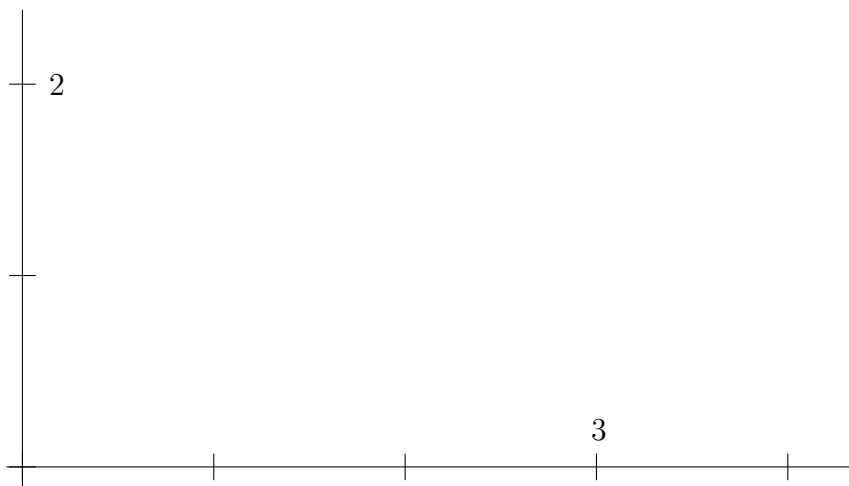
(13) Two populations $x(t)$ and $y(t)$ satisfy:
$$\begin{cases}x' = x(1 - 0.5x - 0.5y) \\y' = y(-0.25 + 0.5x)\end{cases}$$

The critical points are $(0, 0)$, $(2, 0)$ and $(0.5, 1.5)$.

(13a) For each critical point, find the corresponding approximate linear system.

(13b) Classify each critical point as to type and stability.

(13c) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \rightarrow \infty$.



(14) The populations of two species satisfy: $x' = x(1 - 0.5x - 0.5y)$; $y' = y(-0.25 + 0.5x)$.

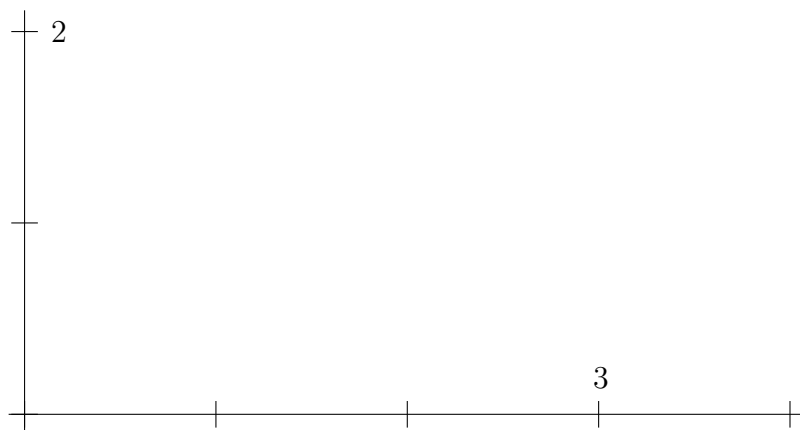
(14a) Find all critical points of the system.

(14b) For each critical point, find the corresponding approx. linear system.

(14c) For each critical point, find the eigenvalues, eigenvectors of the approx. linear system.

(14d) Classify each critical point as to type and stability.

(14e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \rightarrow \infty$, and interpret the results.



(15) The populations of a predator and prey satisfy: $x' = x(1.125 - x - 0.5y)$; $y' = y(-1 + x)$.

(15a) Find all critical points of the system.

(15b) For each critical point, find the corresponding approximating linear system.

(15c) For each critical point, find the eigenvalues, eigenvectors of the approx. linear system.

(15d) Classify each critical point as to type and stability.

(15e) Plot enough of the direction field to determine the behavior of the solutions.