

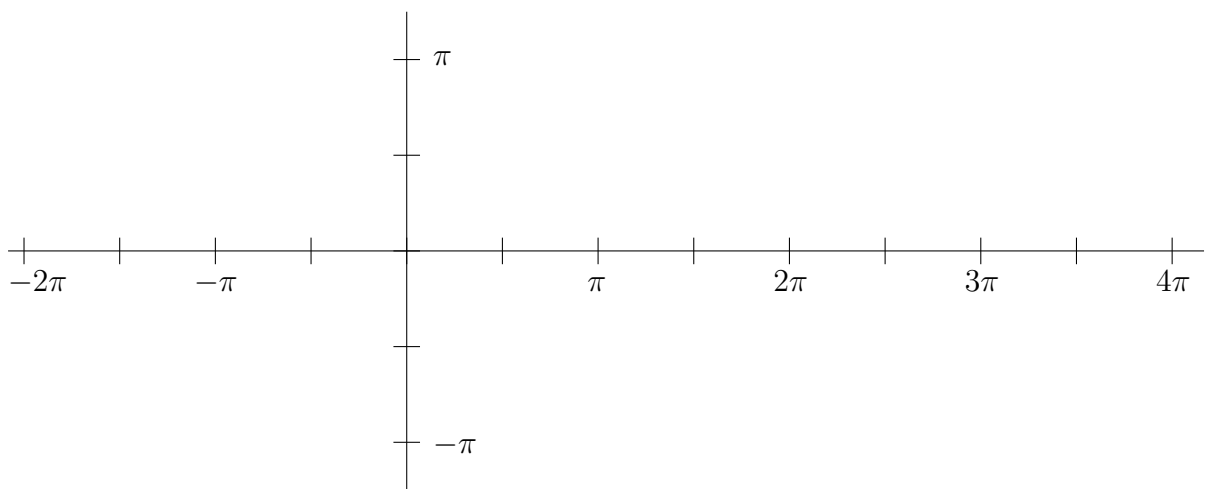
### M309 Sample problems on the Heat Equation

(1) The temperature in a rod of length  $\pi$  satisfies the heat equation  $u_t = 9u_{xx}$ . The temperature initially is  $u(x, 0) = \sin x + \sin 5x$  for  $0 \leq x \leq \pi$ , and the temperature at each end is fixed at 0. Find the temperature  $u(x, t)$  for all  $0 \leq x \leq \pi$ , and all  $t \geq 0$ .

(2a)  $f(x) = \begin{cases} 1, & \text{for } 0 \leq x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \leq x < \pi \end{cases}$

Extend  $f(x)$  as an odd function for  $-\pi \leq x < 0$ . Calculate the Fourier sine series for  $f(x)$ .

(2b) Extend  $f(x)$  periodically with period  $2\pi$ . Sketch the graph of  $f$  for  $-2\pi \leq x \leq 4\pi$ .



(2c) The temperature in a rod of length  $\pi$  satisfies the heat equation  $u_t = 4u_{xx}$ , and the temperature at each end is fixed at 0. The initial temperature is the function  $f(x)$  of (2a) for  $0 \leq x \leq \pi$ . Find the temperature  $u(x, t)$  for all  $0 \leq x \leq \pi$ , and all  $t \geq 0$ .

(3) The temperature in a rod of length  $\pi$  satisfies the heat equation  $u_t = 25u_{xx}$ , and the temperature at each end is fixed at 0. The initial temperature is  $u(x, 0) = \begin{cases} 0, & \text{for } x = 0 \\ 1, & \text{for } 0 < x < \pi \\ 0, & \text{for } x = \pi \end{cases}$

Find the temperature  $u(x, t)$  for all  $0 \leq x \leq \pi$ , and all  $t \geq 0$ .

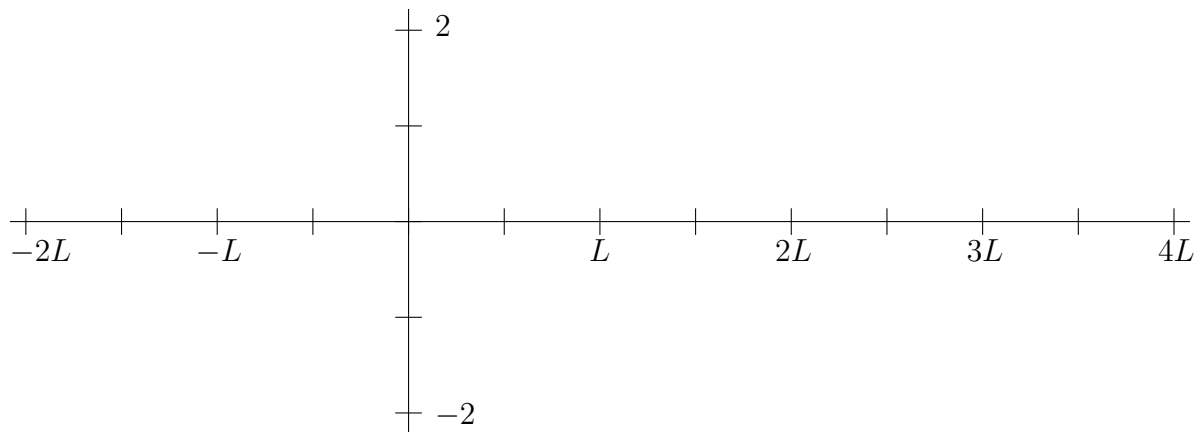
(4) The temperature in a rod of length  $\pi$  satisfies the heat equation  $u_t = 4u_{xx}$ , and the temperature at each end is fixed at 0. The initial temperature is  $u(x, 0) = \begin{cases} 0, & \text{for } 0 \leq x < \frac{\pi}{3} \\ 1, & \text{for } \frac{\pi}{3} \leq x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \leq x < \pi \end{cases}$ . Find the temperature  $u(x, t)$  for all  $0 \leq x \leq \pi$ , and all  $t \geq 0$ .

(5) A function  $f(x)$  is defined by:  $f(x) = \begin{cases} x, & \text{for } 0 \leq x < \frac{L}{2} \\ L - x, & \text{for } \frac{L}{2} \leq x < L \end{cases}$

(5a) Calculate the Fourier sine series for  $f(x)$ . Sketch the graph of  $f(x)$  for  $-2L \leq x \leq 4L$ .

(5b) Calculate the Fourier cosine series for  $f(x)$ . Sketch the graph of  $f(x)$  for  $-2L \leq x \leq 4L$ .

(5c) The temperature in a rod of length  $L$  satisfies the heat equation  $u_t = 9u_{xx}$ , and the temperature at each end is fixed at 0. The initial temperature is  $u(x, 0) = f(x)$ . Find the temperature  $u(x, t)$  for all  $0 \leq x \leq L$ , and all  $t \geq 0$ .



(6) The temperature in a rod of length  $\pi$  satisfies the heat equation  $u_t = 4u_{xx}$ . The temperature initially is  $u(x, 0) = 0$  for  $0 \leq x < \pi$ ,  $u(x, \pi) = \pi$ . The temperature at  $x = 0$  is fixed at 0. The temperature at  $x = \pi$  is fixed at  $\pi$ . Find the temperature  $u(x, t)$  for all  $0 \leq x \leq \pi$ , and all  $t \geq 0$ .