M309A Quiz #1 Oct 6, 1999 Name: ....

(1a) Let $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$, Find the eigenvalues and corresponding eigenvectors of A.

(1b) Find a matrix T and a diagonal matrix D such that $T^{-1}AT = D$.

M309 Quiz #2 Oct 15, 1999 Name: ....

(1a) Let $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$. Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

M309 TEST #3 Oct 25, 1999 Name: ....

(1a) Convert the 2nd order ODE $u'' + u' - 2u = 0$ to a first order system of ODE.'s

(1b) Find the general solution to the system you found in (1a).

(1c) Use the solution of (1b) to find the solution to (1a) with $u(0) = 1$ and $u'(0) = 3$.

(2a) Find the general solution to $x' = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix} x$.

(2b) Classify the origin of the system in (2a) as node, saddle, center or spiral.

(2c) Is system in (2a) as stable or unstable.

(3a) Find the general solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$

(3b) Classify the origin of the system in (3a) as node, saddle, center or spiral.

(3c) Is system in (3a) as stable or unstable.

(3a) Find the solution to $x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x$ with with $x_1(0) = 2$ and $x_2(0) = -4$.

M309 Quiz #4 Nov 3, 1999 Name: ....

The populations of two species satisfy: $x' = x(1 - x + 0.5y)$; $y' = y(2.5 - 1.5y + 0.25x)$.

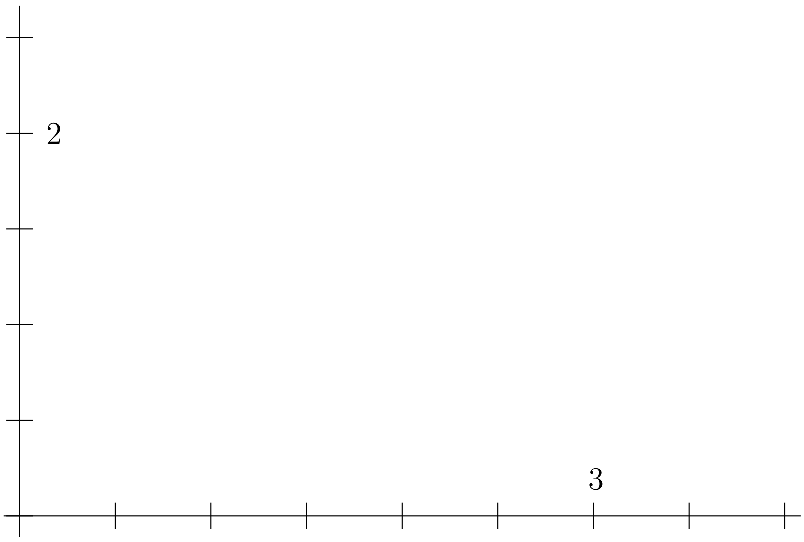
(a) Find all critical points of the system.

(b) For each critical point, find the corresponding approximate linear system.

(c) For each critical point, find the eigenvalues, eigenvectors of the approximate linear system.

(d) Classify each critical point as to type and stability.

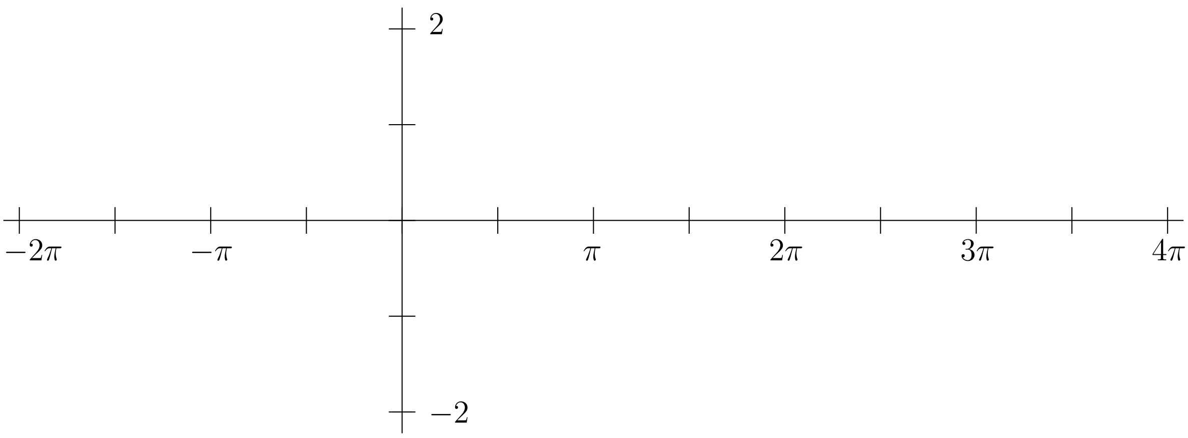
(e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.



M309 Quiz #5 Nov 10, 1997 Name: ....

(1a) A function $f(x)$ is defined by: $f(x) = 1$ for $0 \le x < \pi/2$ $\quad f(x) = 0$ for $\pi/2 \le x < \pi$.

Extend $f(x)$ as an even function for $-\pi \le x \le \pi$. Then extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \le x \le 4\pi$.



(1b) Find the Fourier Series for $f(x)$.

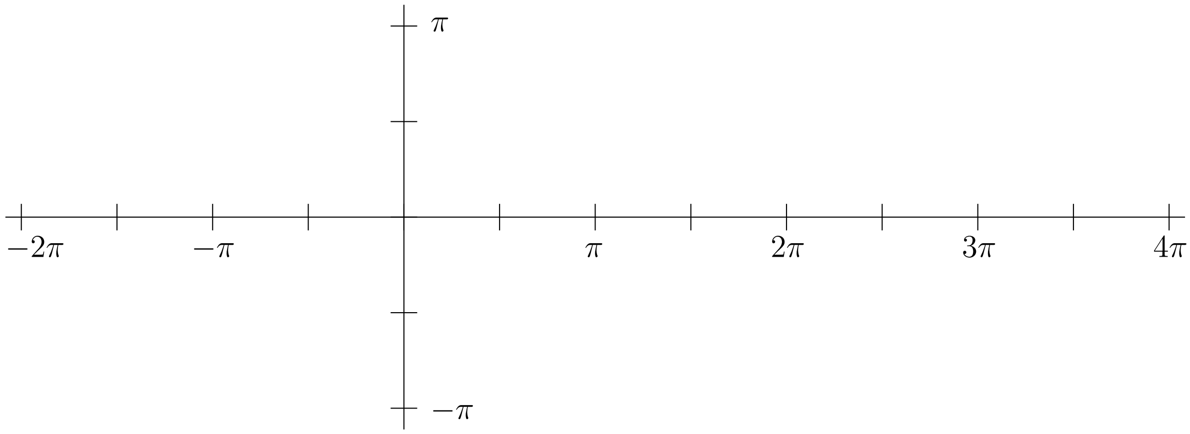
(1c) The temperature in a rod of length π satisfies the heat equation $u_t = u_{xx}$. The ends are insulated, and the temperature initially is $u(x, 0) = f(x)$, the function defined above. Find the temperature $u(x, t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

M309 TEST #6 Nov 19, 1999 Name: ....

(1) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

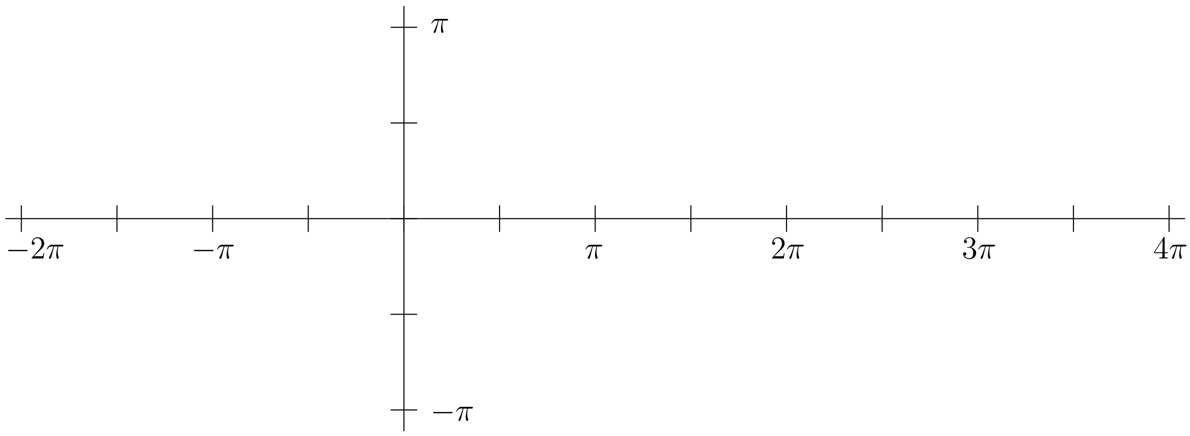
(2) A string in a rod of length π satisfies the wave equation $u_{tt} = 25u_{xx}$. The string is ininially motionless, the ends are fixed at 0. and its initial position is $u(x,0) = \sin x + \sin 3x$ for $0 \le x \le \pi$. Find the position $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(3a) $f(x) = \left\{ \begin{array}{ll} x, & 0 \le x \le \pi/2 \\ \pi - x, & \pi/2 \le x \le \pi \end{array} \right\}$. Extend $f(x)$ as an odd function for $-\pi \le x \le \pi$. Then extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \le x \le 4\pi$.



(3b) Find the Fourier Series for $f(x)$.

(4a) $f(x) = x$, for $0 \le x \le \pi$. Extend $f(x)$ as an even function of period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$.



(4b) Find the Fourier Series for $f(x)$.

M309 TEST Quiz # 7 Dec 3, 1999 Name: ....

(1) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0, y) = 0$, $u(\pi, y) = \sin 3y$ for $0 < y < \pi$ and $u(x, 0) = 0$, $u(x, \pi) = \sin 2x$ for $0 < x < \pi$

(2) Find the solution of Laplace's Equation in the rectangle $0 \le x \le \pi$, $0 \le y \le \pi$, satisfying the boundary conditions: $u(0, y) = 0$, $u(\pi, y) = 0$ for $0 < y < \pi$ and $u(x, 0) = 0$, $u(x, \pi) = 1$ for $0 < x < \pi$

M309 Final Exam #8 Dec 13, 1999 Name: ....

(1) (10 pts) Find the solution $u(r, \theta)$ of Laplace's Equation in the circle $x^2 + y^2 \le 1$ stisfying the boundary condition $f(\theta) = 1 + \sin\theta + \cos 2\theta$.

(2) (20 pts) Find the general solution to $\begin{array}{l} x_1' = x_1 + x_2 + e^t \\ x_2' = 4x_1 - 2x_2 - 2e^t \end{array}$

(3) (10 pts) A string in a rod of length π satisfies the wave equation $u_{tt} = 4u_{xx}$. The string is initially motionless, and its position is given by $u(x, 0) = \sin x + \sin 3x$ for $0 \le x \le \pi$. Find the position $u(x, t)$ for all $0 \le x \le \pi$, and all $t > 0$.

(4) (30 pts) The temperature in a rod of length π satisfies the heat equation $u_t = u_{xx}$. The temperature initially is given by $u(x, 0) = 10$, and the ends are insulated. Find the temperature $u(x, t)$ for all $0 \le x \le \pi$, and all $t > 0$.

(5) (30 pts) $f(x) = \left\{ \begin{array}{ll} 0, & -\pi \le x < -\pi/2 \\ 1, & -\pi/2 \le x < \pi/2 \\ 0, & \pi/2 \le x < \pi \end{array} \right\}$ and $f(x)$ is extended periodically with period 2π.

(5a) Find the Fourier Series for $f(x)$.

(5b) Evaluate the Fourier Series at $x = 0$, and obtain a series for π.

(5c) Write Parseval's equation for $f(x)$ and obtain a series for π^2.