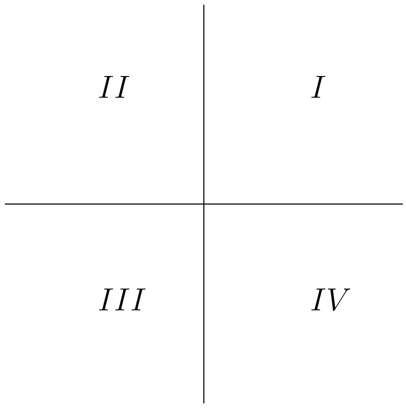
M309 Sample Test #1a

(1a) Let $A = \begin{bmatrix} 3 & 2 \\ -2 & -2 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

(1c) The quadrants are numbered in the diagram below The trajectory of **(1b)** starts in quadrant I. In which quadrant (I, II, III or IV) will the trajectory be when $t = 1000$?



(2a) Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(2b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

(2c) The four quadrants are numbered in the diagram of **(1c)**. The trajectory of **(1b)** starts in quadrant I. What are the next four quadrants through which the trajectory passes?

M309 Sample Test #1b

(1a) Let $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$ Find the general solution to $x' = Ax$.

(1b) Find the solution to $x' = Ax$, with $x(0) = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$.

(2a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

(2b) Find a matrix T and a diagonal matrix D so that $T^{-1}AT = D$.

M309 Sample Test #2a

(1a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & 1 \\ -3 & 2 \end{bmatrix}$.

(1b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(2a) Find the general solution to $\begin{array}{rrcr} x_1' = & 6x_1 & + & x_2 \\ x_2' = & -3x_1 & + & 2x_2 \end{array}$

(2b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

M309 Sample Test #2b

(1a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 7 & 5 \\ -1 & 1 \end{bmatrix}$.

(1b) Find an invertible matrix T and a diagonal matrix D such that $A = TDT^{-1}$.

(2a) Find the general solution to $\begin{array}{rrcr} x_1' = & 7x_1 & + & 5x_2 \\ x_2' = & -x_1 & + & x_2 \end{array}$

(2b) Find the solution to (2a) with $x_1(0) = 1$ and $x_2(0) = 3$.

M309 Sample Test #3a

(1a) Find the general solution to the system $\begin{array}{l} x_1' = x_1 + x_2 \\ x_2' = 4x_1 + x_2 \end{array}$

(1b) Find the solution to the system $\begin{array}{l} x_1' = x_1 + x_2 \\ x_2' = 4x_1 + x_2 \end{array}$ with $x_1(1) = 3$, $x_2(1) = 2$.

(2) Find the general solution to the system $\begin{array}{l} x_1' = x_1 + x_2 + e^t \\ x_2' = 4x_1 + x_2 + -2e^t \end{array}$

M309 Sample Test #3b

(1) Section 9.4 # 4: Find all critical points of the system: $\begin{cases} x' = x(1.5 - 0.5x - y) \\ y' = y(0.75 - y - 0.125x) \end{cases}$

(2) (Section 9.2 # 11:) $(0.5, 0.5)$ is a critical point of the system $\begin{cases} x' = -x + 2xy \\ y' = y - x^2 - y^2 \end{cases}$

For the critical point $(0.5, 0.5)$ find the corresponding approximate linear system and find the eigenvalues and eigenvectors. Classify the critical point $(0.5, 0.5)$ by type, and determine whether or not it is **AS** (asymptotically stable), **S** but not **AS** (stable but not asymptotically stable), or **US** (unstable).

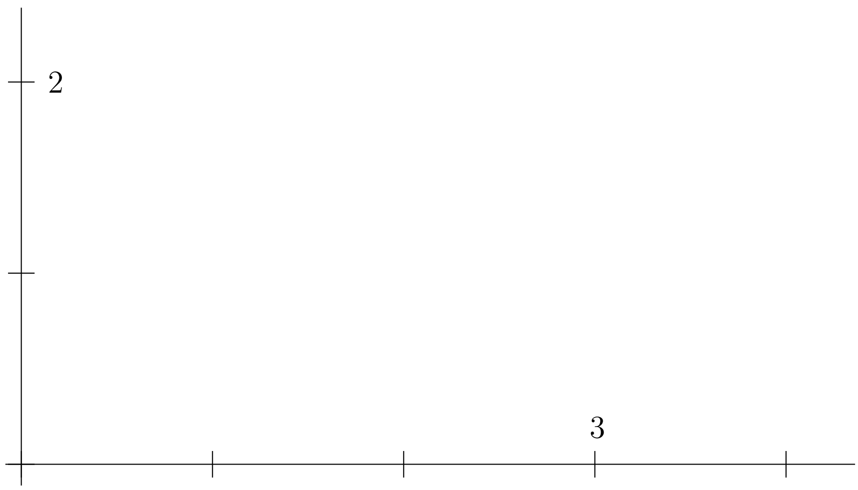
(4) Two populations $x(t)$ and $y(t)$ satisfy: $\begin{cases} x' = x(1 - 0.5x - 0.5y) \\ y' = y(-0.25 + 0.5x) \end{cases}$

The critical points are $(0, 0)$, $(2, 0)$ and $(0.5, 1.5)$.

(a) For each critical point, find the corresponding approximate linear system.

(b) Classify each critical point as to type and stability.

(c) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$.



M309 Sample Test #4a

We want an approximate solution to $\begin{aligned} x' &= x(1.5 - 0.5y) \\ y' &= y(-0.5 + x) \end{aligned}$ with $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}$.

(1) Take $\Delta t = 0.2$ and find approx. values for $(x(0.2), y(0.2))$.

(2) Do two steps, each with $\Delta t = 0.1$, and find approx. values for $(x(0.2), y(0.2))$.

(3) The populations of two species satisfy: $x' = x(1 - 0.5x - 0.5y)$; $\quad y' = y(-0.25 + 0.5x)$.

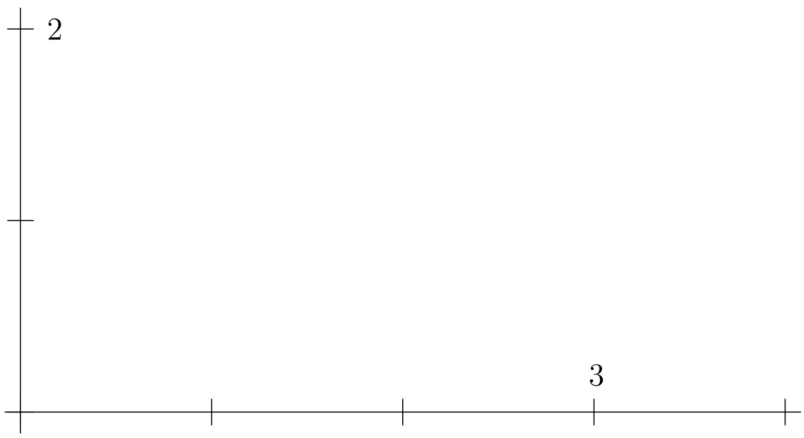
(a) Find all critical points of the system.

(b) For each critical point, find the corresponding approx. linear system.

(c) For each critical point, find the eigenvalues, eigenvectors of the approx. linear system.

(d) Classify each critical point as to type and stability.

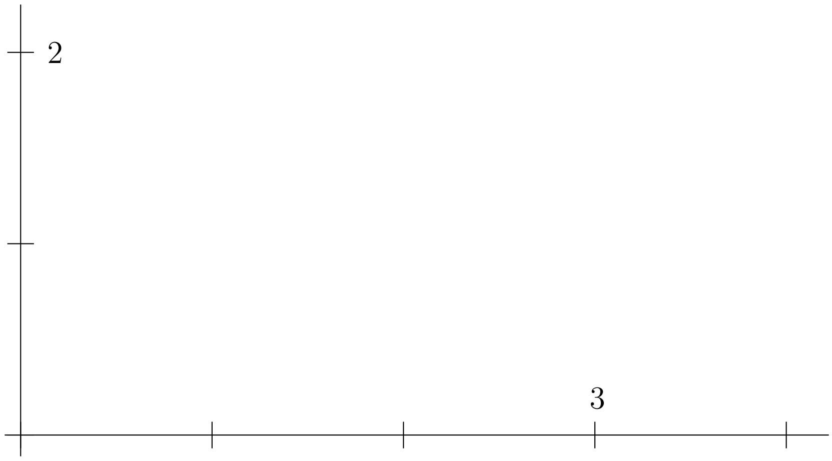
(e) Plot enough of the direction field to determine the behavior of the solutions. Determine the limiting behavior of $(x(t), y(t))$ as $t \longrightarrow \infty$, and interpret the results.



M309 Sample Test #4b

We want an approximate solution to $\begin{aligned} x' &= x(1-0.5y) \\ y' &= y(-.75+0.25x) \end{aligned}$ with $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(1) Do two steps, each with $\Delta t = 0.5$, and find approx. values for $(x(1), y(1))$. Plot the points $(x(0.0), y(0.0))$ $(x(0.5), y(0.5))$ $(x(1), y(1))$ on the graph below.



(2) The populations of a predator and prey satisfy: $x' = x(1.125 - x - 0.5y)$; $y' = y(-1+x)$.

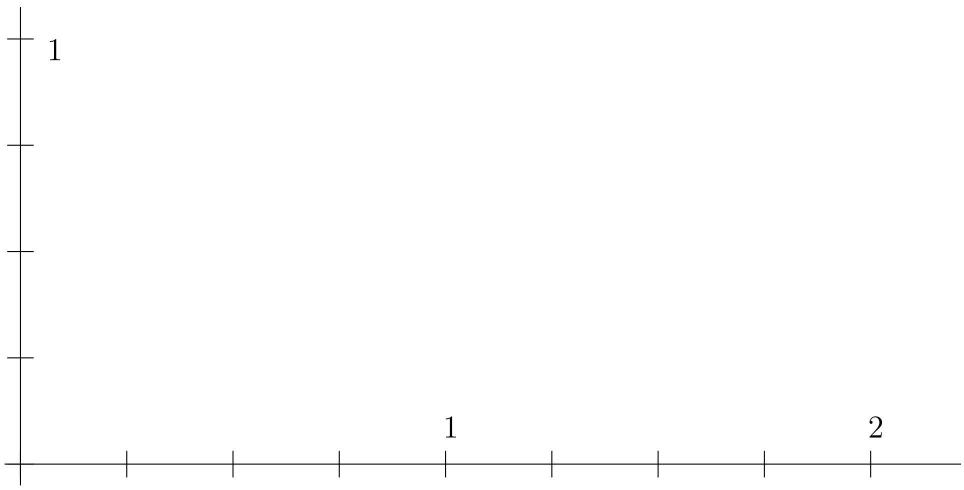
(a) Find all critical points of the system.

(b) For each critical point, find the corresponding approximating linear system.

(c) For each critical point, find the eigenvalues, eigenvectors of the approx. linear system.

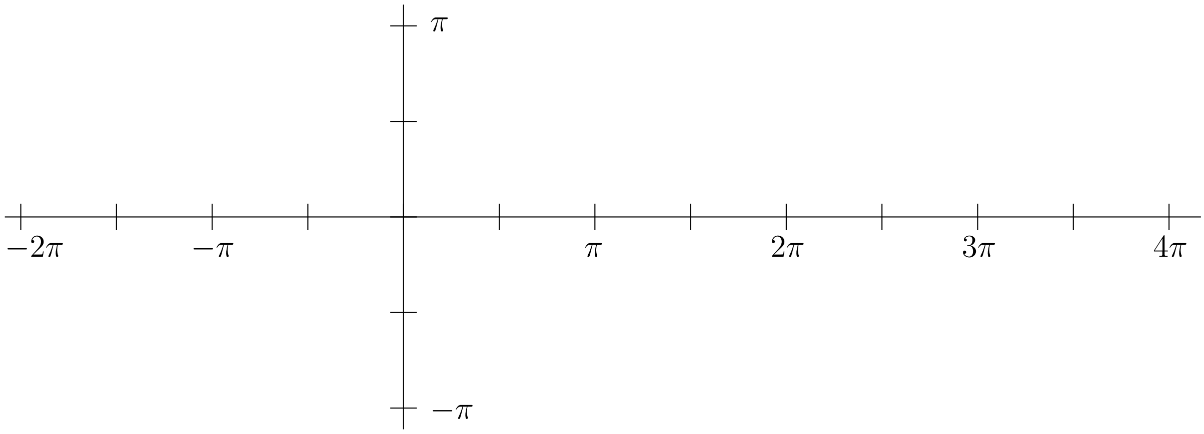
(d) Classify each critical point as to type and stability.

(e) Plot enough of the direction field to determine the behavior of the solutions.

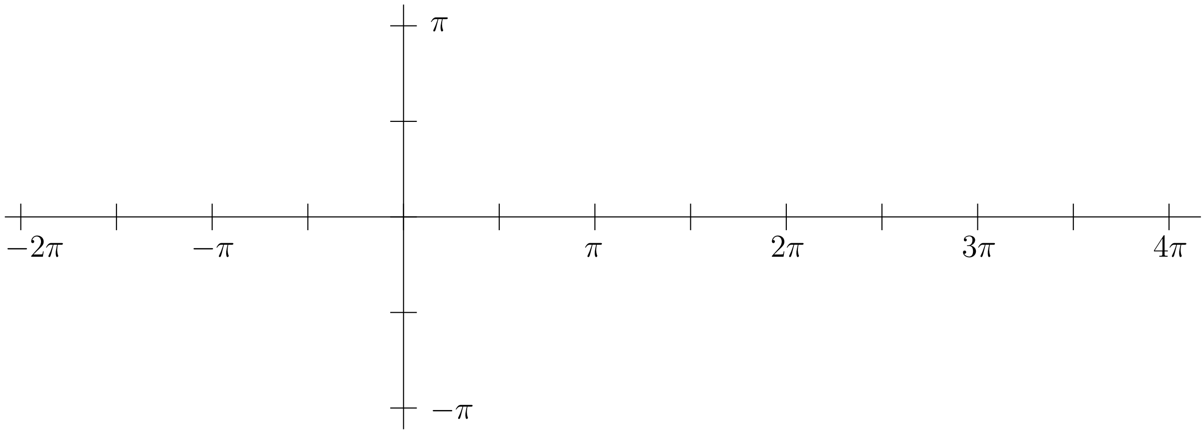


M309 Sample Test #5a May 19, 2001 Name: ....

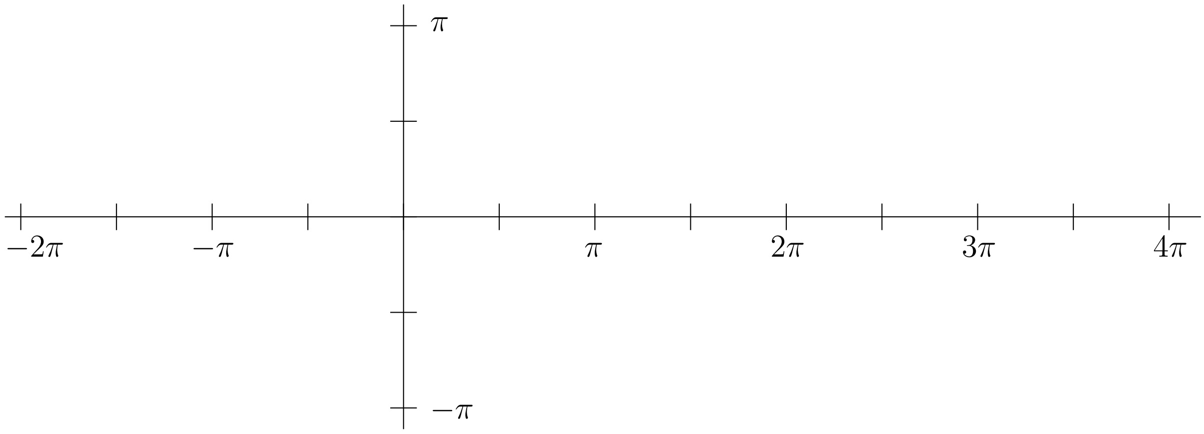
(1) $f(x) = \pi - x$, for $0 \le x \le \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Calculate the Fourier Series for $f(x)$.



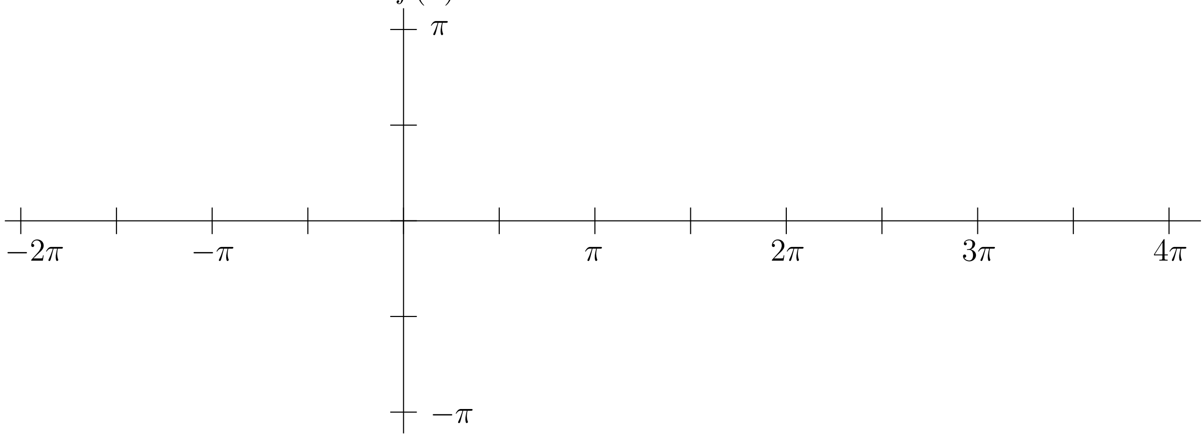
(2) $f(x) = x$, for $0 \le x \le \frac{\pi}{2}$ and $f(x) = \pi - x$, for $\frac{\pi}{2} \le x \le \pi$ Extend $f(x)$ as an <u>odd</u> function of period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Calculate the F.S. for $f(x)$.



(3) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ 1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$.

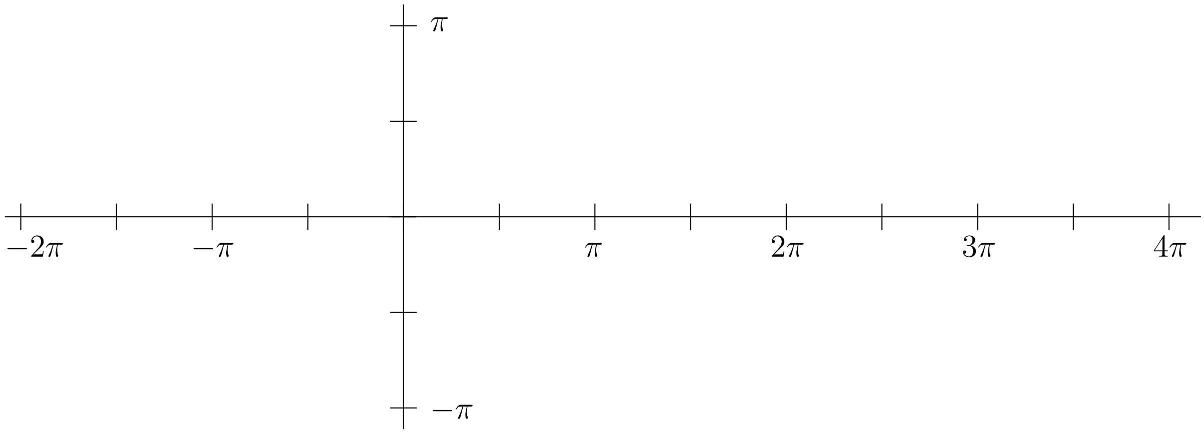


(4) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$. ß



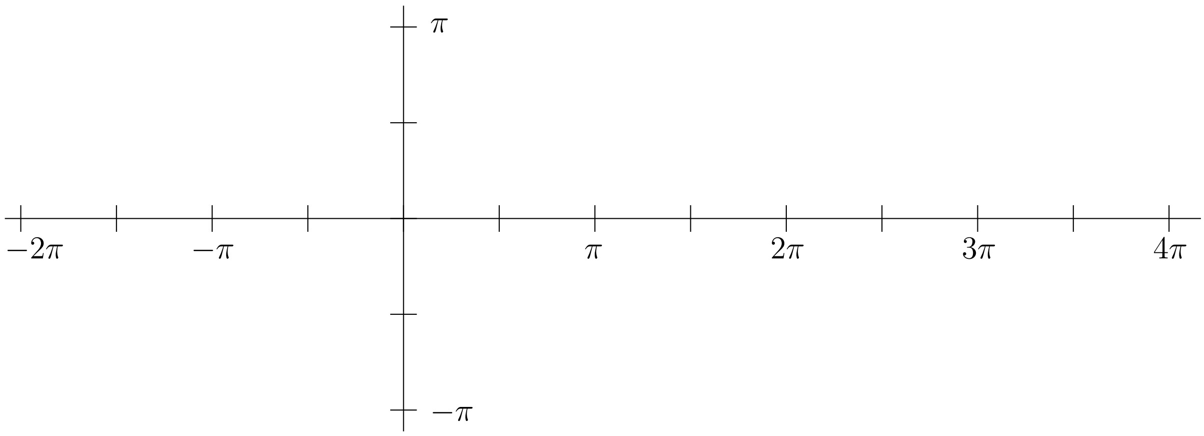
(5) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ |x|, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π.

Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$.



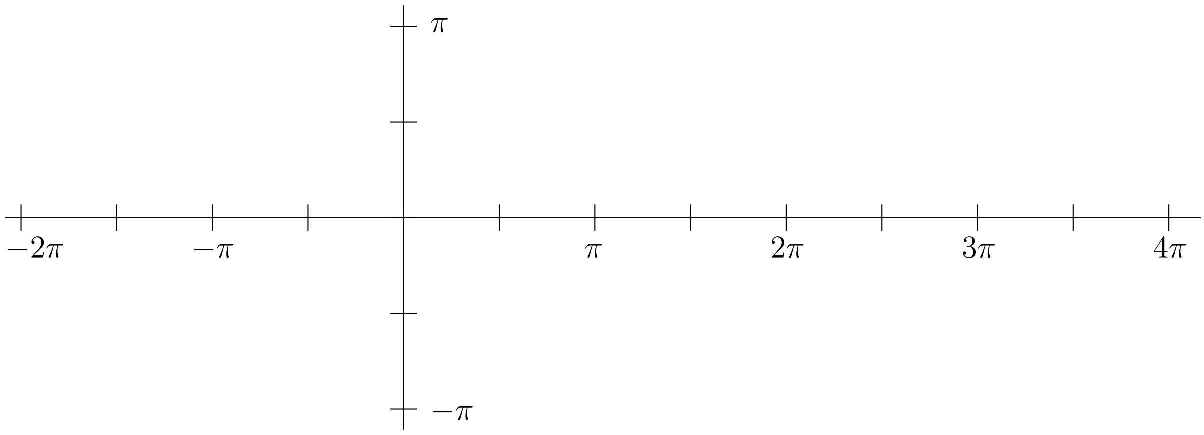
(6) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ x+1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π.

Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$.

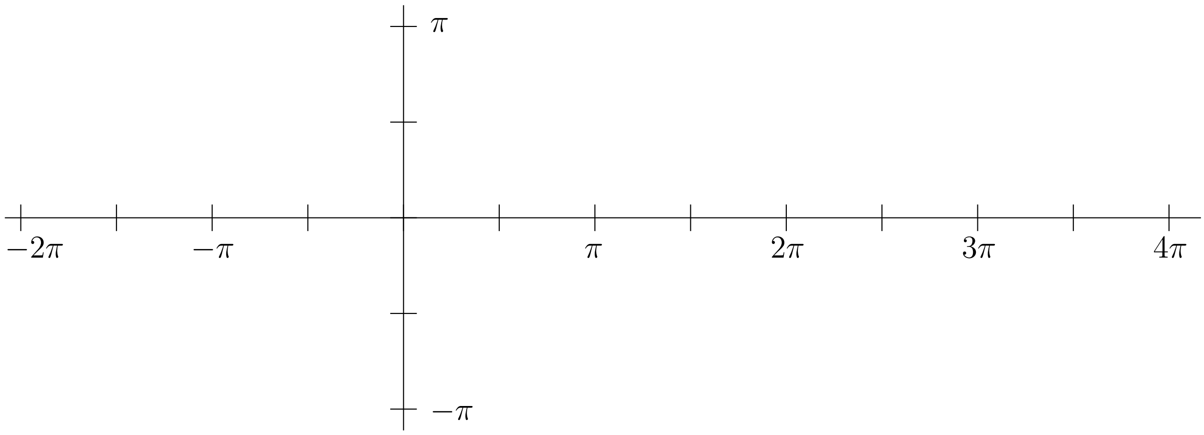


M309 Sample Test #5b

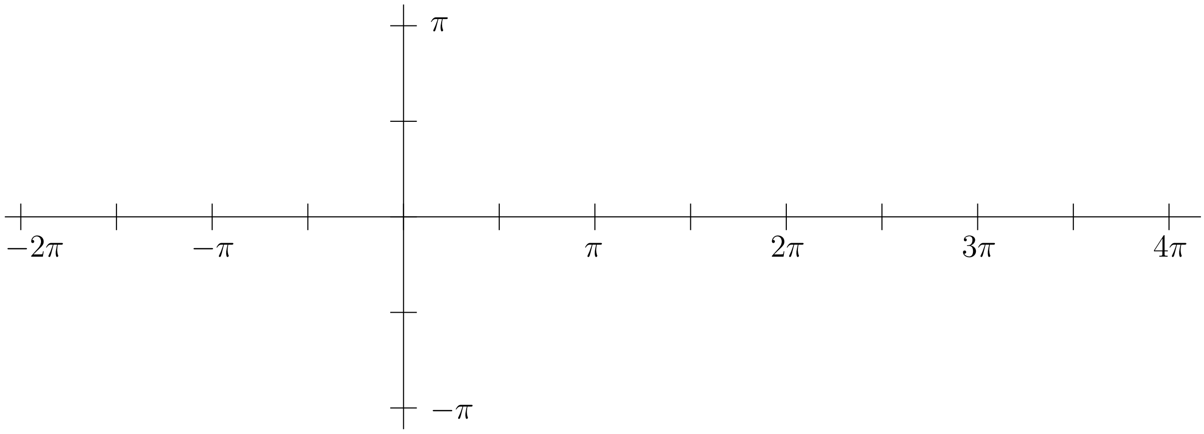
(1) $f(x) = \pi - x$, for $0 \leq x \leq \pi$. Extend $f(x)$ as an <u>even</u> function of period 2π. Sketch the graph of f for $-2\pi \leq x \leq 4\pi$.



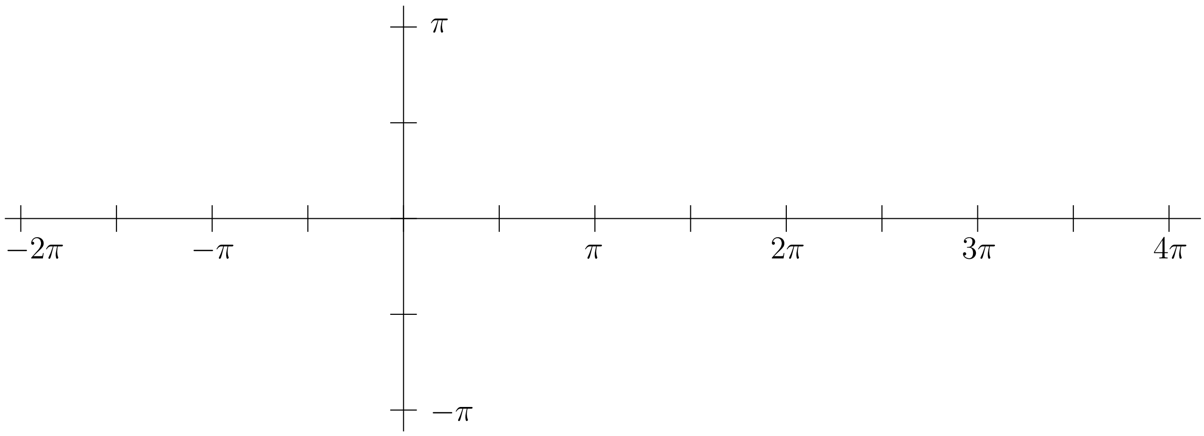
(2) $f(x) = x$, for $0 \leq x \leq \dfrac{\pi}{2}$ and $f(x) = \pi - x$, for $\dfrac{\pi}{2} \leq x \leq \pi$ Extend $f(x)$ as an <u>odd</u> function of period 2π. Sketch the graph of f for $-2\pi \leq x \leq 4\pi$.



(3) $f(x) = \begin{cases} 0, & \text{for } -\pi \le x < -\frac{\pi}{2} \\ |x|+1, & \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$; $f(x)$ is extended periodically with period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither?



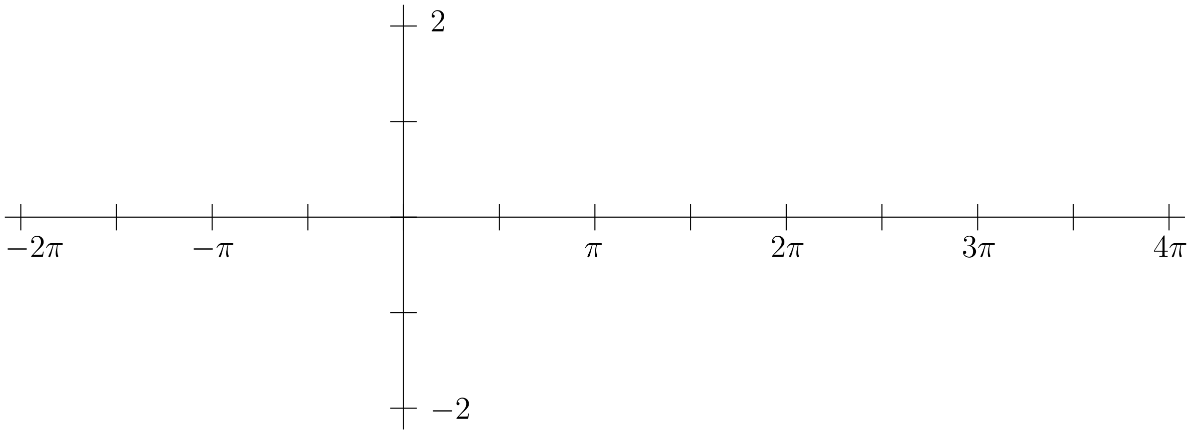
(4) $f(x) = \begin{cases} 0, & \text{for } -\pi < x < -\frac{\pi}{2} \\ 1, & \text{for } -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ 0, & \text{for } \frac{\pi}{2} < x \le \pi \end{cases}$; $f(x)$ is extended periodically with period 2π. Sketch the graph of f for $-2\pi \le x \le 4\pi$. Is $f(x)$ an even function, an odd function, or neither? Calculate the Fourier Series for $f(x)$.



M309 Sample Test #5c

(1) The temperature in a rod of length π satisfies the heat equation $u_t = 9u_{xx}$. The temperature initially is $u(x,0) = \sin 2x + \sin 5x$ for $0 \le x \le \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

(2) A function $f(x)$ is defined by: $f(x) = x$ for $0 \le x < \pi/2$ and $f(x) = \pi - x$ for $\pi/2 \le x < \pi$. Extend $f(x)$ as an even function for $-\pi \le x < \pi$. Extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \le x \le 4\pi$.



Calculate the Fourier Series for $f(x)$.

(3) The temperature in a rod of length π satisfies the heat equation $u_t = 4u_{xx}$, and the temperature at each end is fixed at 0. The initial temperature is $u(x,0) = \begin{cases} 1, & \text{for } 0 \le x < \frac{\pi}{2} \\ -1, & \text{for } \frac{\pi}{2} \le x < \pi \end{cases}$

Find the temperature $u(x,t)$ for all $0 \le x \le \pi$, and all $t \ge 0$.

M309 Sample Final Exam

(1) A string length π satisfies the wave equation $u_{tt} = 9u_{xx}$. The string is initially motionless, the ends are fixed at 0, and its initial position is $u(x, 0) = \sin x + \sin 2x$ for $0 \leq x \leq \pi$. Find the position $u(x, t)$ for all $0 \leq x \leq \pi$, and all $t \geq 0$.

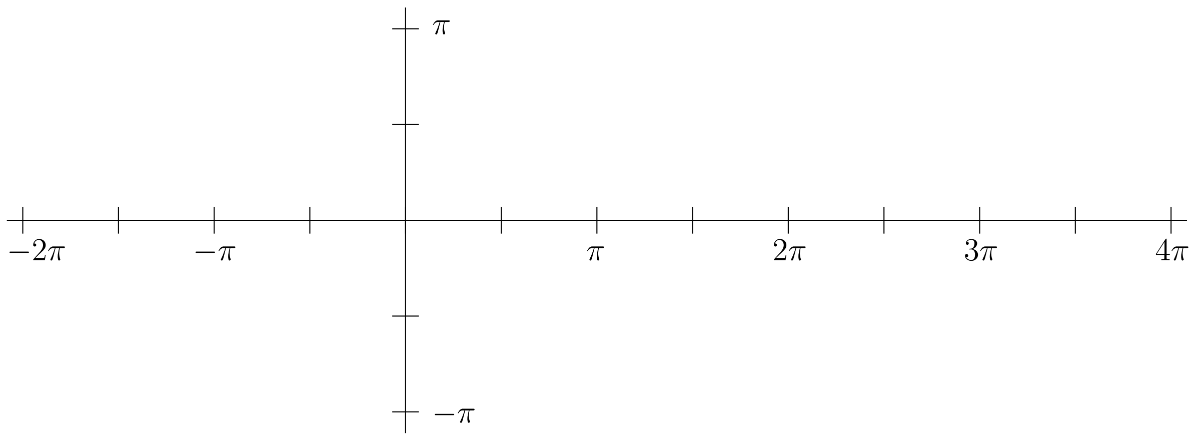
(2) The temperature in a rod of length π satisfies the heat equation $u_t = 16u_{xx}$. The temperature initially is $u(x, 0) = \sin x + \sin 3x$ for $0 \leq x \leq \pi$, and the temperature at each end is fixed at 0. Find the temperature $u(x, t)$ for all $0 \leq x \leq \pi$, and all $t \geq 0$.

(3) Find the solution of Laplace's Equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$, satisfying the boundary conditions:

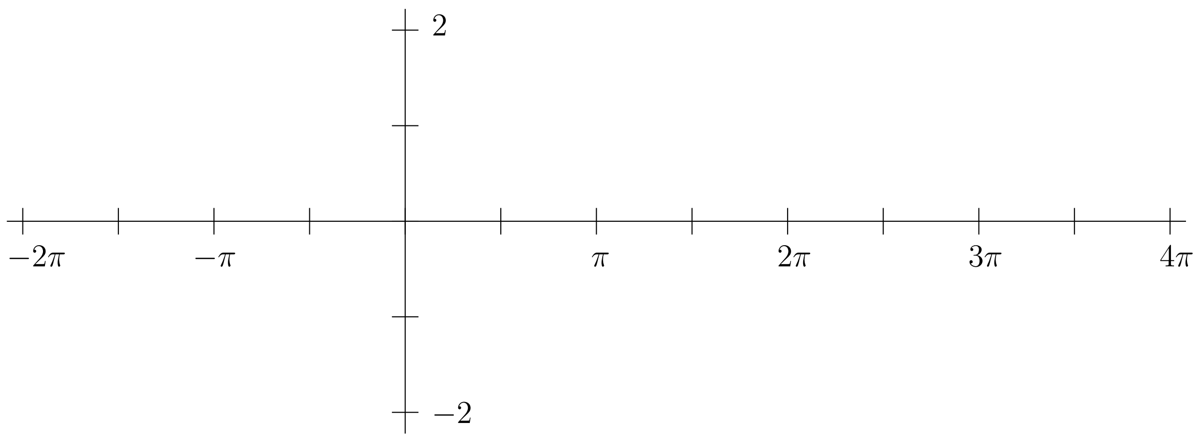
$$u(0, y) = 0, \quad \text{for } 0 < y < \pi; \qquad u(a, y) = \sin \frac{\pi y}{b} \quad \text{for } 0 < y < b;$$

$$u(x, 0) = 0 \quad \text{for } 0 < x < \pi; \qquad u(x, b) = \sin \frac{2\pi x}{a} \quad \text{for } 0 < x < a.$$

(4) A function $f(x)$ is defined by: $f(x) = x$ for $0 \leq x < \pi/2$ and $f(x) = \pi - x$ for $\pi/2 \leq x < \pi$. Extend $f(x)$ as an <u>odd</u> function for $-\pi \leq x < \pi$. Extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \leq x \leq 4\pi$. Calculate the Fourier Series for $f(x)$.



(5) A function $f(x)$ is defined by: $f(x) = 1$ for $0 \leq x < \pi$ and $f(x) = 0$ for $\pi \leq x < 2\pi$. Extend $f(x)$ periodically with period 2π. Sketch the graph of $f(x)$ for $-2\pi \leq x \leq 4\pi$.



Calculate the Fourier Series for $f(x)$.