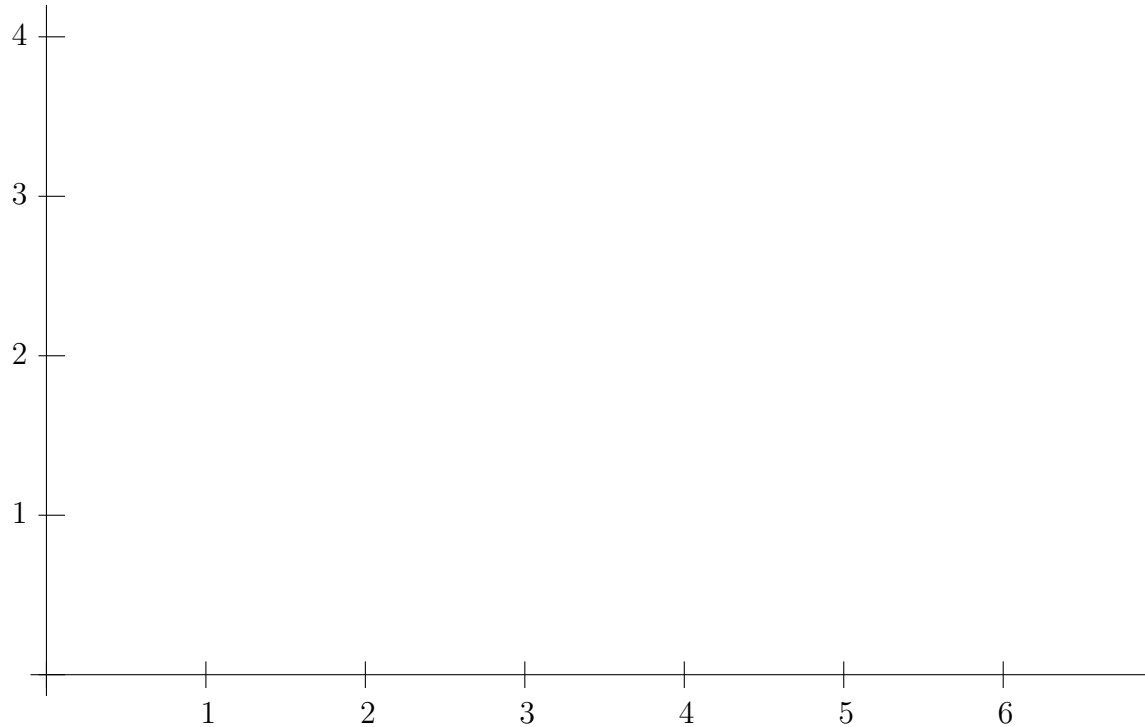


Sample Quiz 3 for M309

- (1) Two non-negative populations $x(t)$, $y(t)$ satisfy $\begin{cases} x' = x(4 - x - y) \\ y' = y(6 - 2y - x) \end{cases}$ for $x \geq 0$, $y \geq 0$.

The lines $x + y = 4$ and $x + 2y = 6$ separate the positive quadrant into four regions. Give the signs of the components of direction field in each of the four regions. Sketch a vector of the direction field in each region.

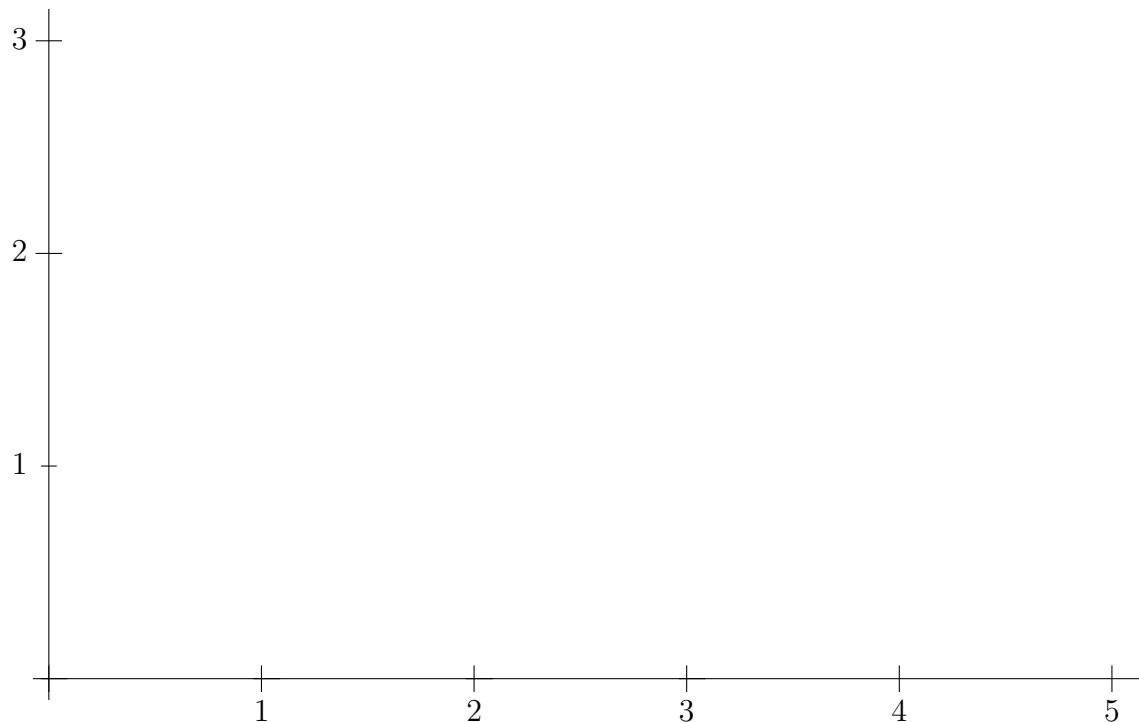


- (1b) The point $(2, 2)$ is a critical point of (1a). Find the approximating linear system near $(2, 2)$. What is the type and stability of $(2, 2)$?

(2) Two non-negative populations $x(t)$ and $y(t)$ satisfy
$$\begin{cases} x' = x(5 - y - x) \\ y' = y(-3 + x) \end{cases},$$

(2a) The point $(3, 2)$ is a critical point. What is its type and stability?

(2b) Plot the direction field at each of the points $(4, 2)$, $(3, 1)$, $(2, 2)$, $(3, 3)$,



(3) Non-negative populations $x(t)$ and $y(t)$ satisfy
$$\begin{cases} x' = x(4 - x - 2y) \\ y' = y(-3 + 2x - y) \end{cases} \text{ for } x \geq 0, y \geq 0.$$

Find the critical points of this system, and for each critical point give its type and stability