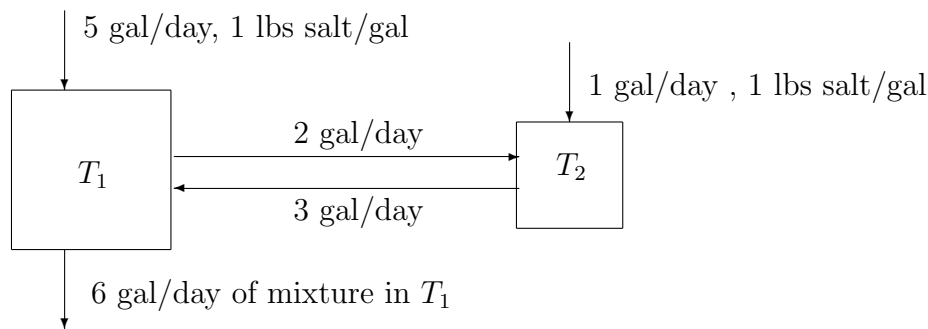


M309 Quiz # 1 Solutions

(1a) There are two tanks, with pipes carrying salt water as shown:



T_1 holds 10 gallons of salt water; T_2 holds 5 gallons of salt water.

Let Q_1 be pounds of salt in T_1 at time t ;

Let Q_2 be pounds of salt in T_2 at time t ;

Write a system of differential equations for Q_1 and Q_2 .

Solution:

$$\begin{aligned} Q_1' &= -0.8Q_1 + 0.6Q_2 + 5 \\ Q_2' &= 0.2Q_1 - 0.6Q_2 + 1 \end{aligned}$$

(2) Find the solution to $\begin{cases} x_1' = 2x_1 + x_2 \\ x_2' = 2x_1 + 3x_2 \end{cases}$ where $x(0) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Solution: $\det A - rI = \det \begin{bmatrix} 2-r & 1 \\ 2 & 3-r \end{bmatrix} = r^2 - 5r + 4 = (r-1)(r-4)$

$$r = 1, A - I = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \quad a_1 + a_2 = 0, \quad \xi = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$r = 4, A - I = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \quad -2a_1 + a_2 = 0, \quad \xi = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The general solution is $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$

$$\begin{aligned} C_1 + C_2 &= 5 \\ -C_1 + 2C_2 &= 4 \end{aligned} \quad 3C_2 = 9; \quad C_2 = 3; \quad C_1 = 2.$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$$