Due Wednesday, June 8 at 5:00 PM in my office (C-554) or my mailbox. No late papers will be accepted (except for dire emergencies and with prior arrangement).

Consultation with other persons — or the internet — is strictly prohibited. You may only use your book and lecture notes, or these materials from a previous course. If you use a result from a previous course, you must say where the result came from, unless it is “well-known”. If you are not sure about what is allowed, ask me. Not understanding the rules will not be an acceptable excuse.

Each problem is worth 10 points.

1. Exercise 21 of 3-3 in do Carmo.

2. Let \( M \) be a compact oriented regular surface with orientation \( N \). The work done in Section 2-7 of do Carmo implies that if \( |\epsilon| > 0 \) is sufficiently small, then

\[
\overline{M} = \{ p + \epsilon N(p) : p \in M \}
\]

is a regular surface and the map \( M \to \overline{M} \) sending \( p \) to \( p + \epsilon N(p) \) is a diffeomorphism. (You need not prove this.)

(a) Prove that \( N(p) \) is normal to \( \overline{M} \) at the point \( p + \epsilon N(p) \).

(b) Prove that the principal curvatures \( \kappa_i \) of \( \overline{M} \) at \( p + \epsilon N(p) \) are given by

\[
\kappa_i = \frac{k_i(p)}{1 - \epsilon k_i(p)}
\]

where \( k_i(p) \) denote the principal curvatures of \( M \) at \( p \).

3. Let \( S \) be a regular surface and \( \mathbf{x} : U \to S \) be a parametrization, with \( \mathbf{x}(U) = S \) and \( U \) open in \( \mathbb{R}^2 \). Suppose that the covariant derivative \( D\mathbf{x}/dt \) is identically 0 along every parametrized curve \( \alpha \) in \( S \). Prove that the Gaussian curvature of \( S \) is everywhere 0.

4. Problem 23 of 4-4 in do Carmo.