

Math 443
Spring 2008
Homework 2

Read Section 3-3 and pages 197–203 in do Carmo. Then do exercises 4, 6ac, 7e, 13, 16 from 3-3 and exercise 13 from 3-5.

Remarks and modifications to the above exercises:

1. In exercise 4 of 3-3, you will need to know that

$$\int \frac{dx}{(1+x^2)^{\frac{1}{2}}} = \sinh^{-1} x + C,$$

where \sinh^{-1} is the inverse of \sinh , and where

$$\sinh x = \frac{e^x - e^{-x}}{2}.$$

2. In exercise 6 of 3-3, take r to be the z -axis and a normal to r (in the plane of the curve) as the x -axis. Restrict attention to the part of C lying in the first quadrant, so that C is the graph of a decreasing function z of x , $0 < x \leq 1$. Note that the line tangent to C at $x = 1$ must be parallel to the x -axis; for definiteness, take this line as the x -axis. For part c, rotate the part of the tractrix lying in the first quadrant with $0 < x < 1$ about the z -axis to obtain a regular surface; prove that the Gaussian curvature is identically -1 .

3. You already proved (exercise 17 of 3-2) one direction of exercise 13a of 3-5. Prove the other direction. In part b, let $\mathbf{x} : U \rightarrow S^2$ be any isothermal parametrization; for example, \mathbf{x} could be the inverse of stereographic projection.

This assignment is due Wednesday, April 23.