(1) Let $\langle\ ,\ \rangle$ be a positive definite inner product on the vector space V. Let $L : V \to V$ be a linear operator that satisfies the condition

$$\langle u, L(v)\rangle = \langle L(u), v\rangle \text{ for all } u, v, \in V\,.$$

Finally, let v_λ and v_μ be eigenvectors associated to the two eigenvalues λ and μ, respectively. Prove that if $\lambda \neq \mu$, then $v_\lambda \perp v_\mu$.

(2) Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, and $P_3 = (x_3, y_3)$ be three non-colinear points in $\mathbb{R}^2$. Show that the equation of the circle passing through these points is given by the equation

$$\begin{vmatrix} 1 & x & y & x^2+y^2 \\ 1 & x_1 & y_1 & x_1^2+y_1^2 \\ 1 & x_2 & y_2 & x_2^2+y_2^2 \\ 1 & x_3 & y_3 & x_3^2+y_3^2 \end{vmatrix} = 0$$

(3) Let n be odd and let A be an $n \times n$ skew-symmetric matrix.

(a) Prove that $\det(A) = 0$.

(b) Prove that the map $L_A : \mathbb{R}^n \to \mathbb{R}^n$ is not surjective.