

Print Your Name

Signature

Student ID Number

Quiz Section

Professor's Name

TA's Name

**!!! READ...INSTRUCTIONS...READ !!!**

1. Your exam contains 9 questions and 11 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. The entire exam is worth 100 points. Point values for problems vary and these are clearly indicated. You have 2 hours and 50 minutes for this final exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Make sure to do your own work on the exam.
4. There is plenty of space on the exam to do your work. If you need extra space, use the back pages of the exam and clearly indicate this.
5. You are allowed one  $8.5 \times 11$  sheet of handwritten notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed. Make sure your calculator is in radian mode.
6. Unless otherwise instructed, ALWAYS GIVE YOUR ANSWERS IN EXACT FORM. For example,  $3\pi$ ,  $\sqrt{2}$ ,  $\ln(2)$  are in exact form; the corresponding approximations 9.424778, 1.4142, 0.693147 are NOT in exact form.

Problem	Total Points	Score
1	12	
2	9	
3	12	
4	10	
5	12	

Problem	Total Points	Score
6	12	
7	12	
8	12	
9	9	
Total	100	

1. (12 points) Find the derivatives of the following functions. You do not have to simplify.

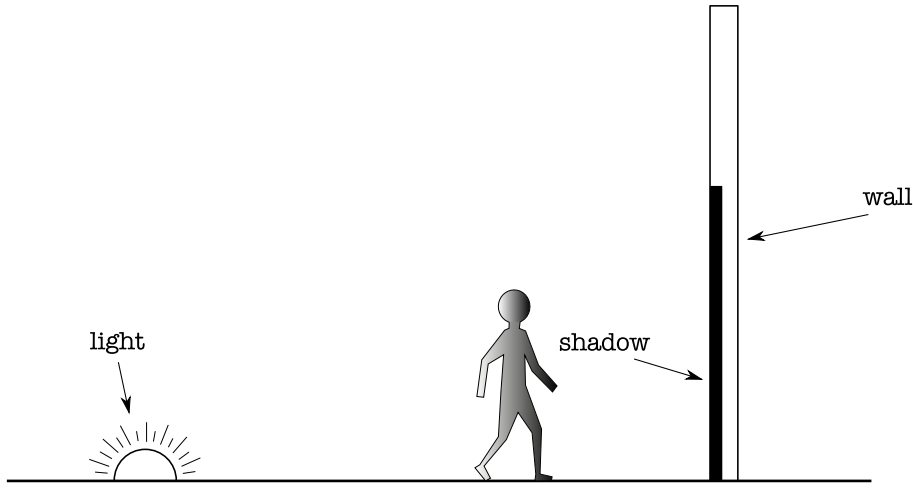
(a)  $\frac{\sec x}{1 + x^4}$

(b)  $2^x \tan(3x)$

(c)  $(1 + e^x)^x$

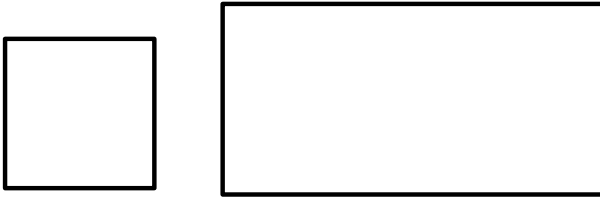
(d)  $\sin^{-1}(\sqrt{4 - x^2})$

2. (9 points) A spotlight on the ground shines on a wall 40 feet away. A woman 6 feet tall walks from the spotlight toward the wall at a speed of 5 feet/sec. How fast is the height of her shadow on the wall decreasing when she is 10 feet away from the wall?



3. (12 points) A farmer has 136 meters of fencing. She wants to make two rectangular enclosures. One will be square. The other will have its long side twice as long as its short side. (Allow the possibility that all of the fencing could go to only one of the enclosures.)

For instance, the enclosures might look like this:



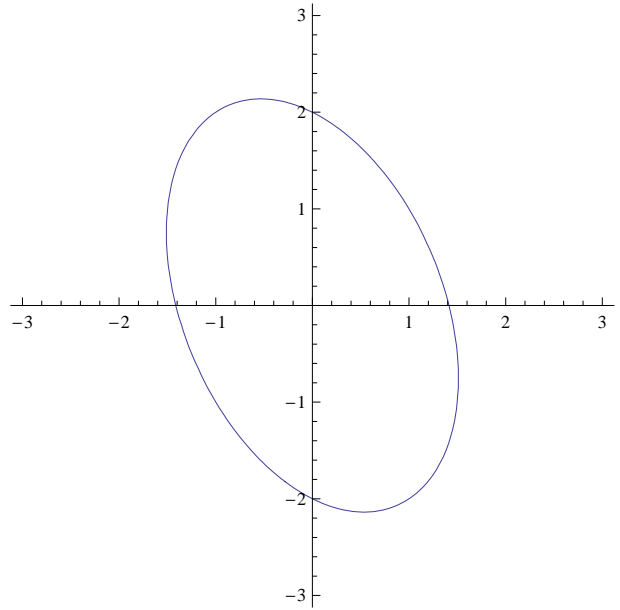
- (a) What should the dimensions of the enclosures be to make the combined total area of the enclosures as **small** as possible?

- (b) What should the dimensions of the enclosures be to make the combined total area of the enclosures as **large** as possible?

4. (10 points)

The pictured curve is given by the equation

$$2x^2 + xy + y^2 = 4$$



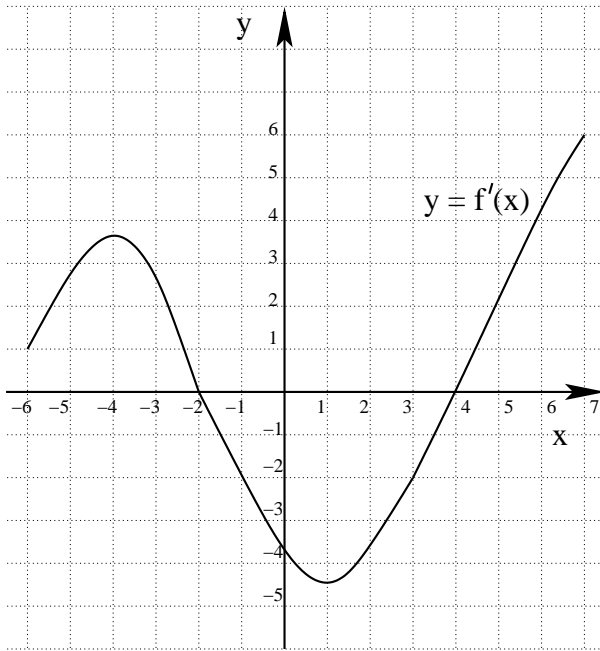
(a) Compute the derivative  $\frac{dy}{dx}$  implicitly.

(b) Find the equation of the tangent line at the point  $(0, 2)$ .

(c) Find the equation of the tangent line at the point  $(\frac{4\sqrt{7}}{7}, -\frac{2\sqrt{7}}{7})$ .

5. (12 points) In the figure below, you have the graph of  $y = f'(x)$ , the **DERIVATIVE** of a certain (unknown) function  $f(x)$ , on the interval  $-6 \leq x \leq 7$ .

Based on this graph, answer the following questions. The numbers in all answers should be integers.



- (a) At what value(s) of  $x$  does  $f(x)$  have a local maximum?
- (b) At what value(s) of  $x$  does  $f(x)$  have a local minimum?
- (c) At what value(s) of  $x$  is  $f''(x) = 0$ ?
- (d) List the  $x$ -interval(s) on which  $f$  is increasing.
- (e) List the  $x$ -interval(s) on which  $f$  is concave down.

6. (12 points) An object is moving back and forth on the  $x$ -axis according to the equation  $x = 3 \sin(20\pi t)$ ,  $t \geq 0$ , where  $x$  is measured in cm and  $t$  in seconds. In this problem leave all answers in exact form and include the correct units.

(a) How many complete back-and-forth motions (from the origin to the right, then to the left, and then back to the origin) does the object make in one second?

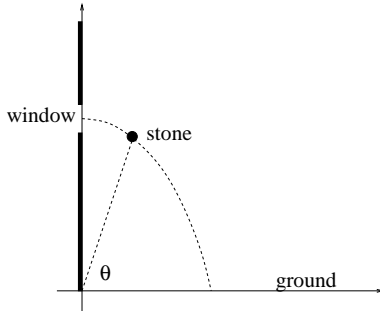
(b) What is  $t$  the first time that the object is at its farthest right?

(c) At the time found in part (b), what is the object's velocity?

(d) At the time found in part (b), what is the object's acceleration?

7. (12 points) A stone is thrown at 4 ft/sec from a window that is 180 feet above the ground (as pictured below). Its trajectory is given by the parametric equations

$$x = 4t; \quad y = 180 - 16t^2.$$



- (a) Let  $\theta$  be the angle in radians above the horizontal of the line of sight from the origin to the stone. Write a formula for  $\theta$  as a function of  $t$ .

- (b) Find the instantaneous rate of change of  $\theta$  when  $t = 3$  seconds.

8. (12 points) Consider the graph of the function

$$f(x) = \frac{4x}{x^2 + 1}.$$

(a) Find all vertical asymptotes or state that there are none.

(b) Find all horizontal asymptotes or state that there are none.

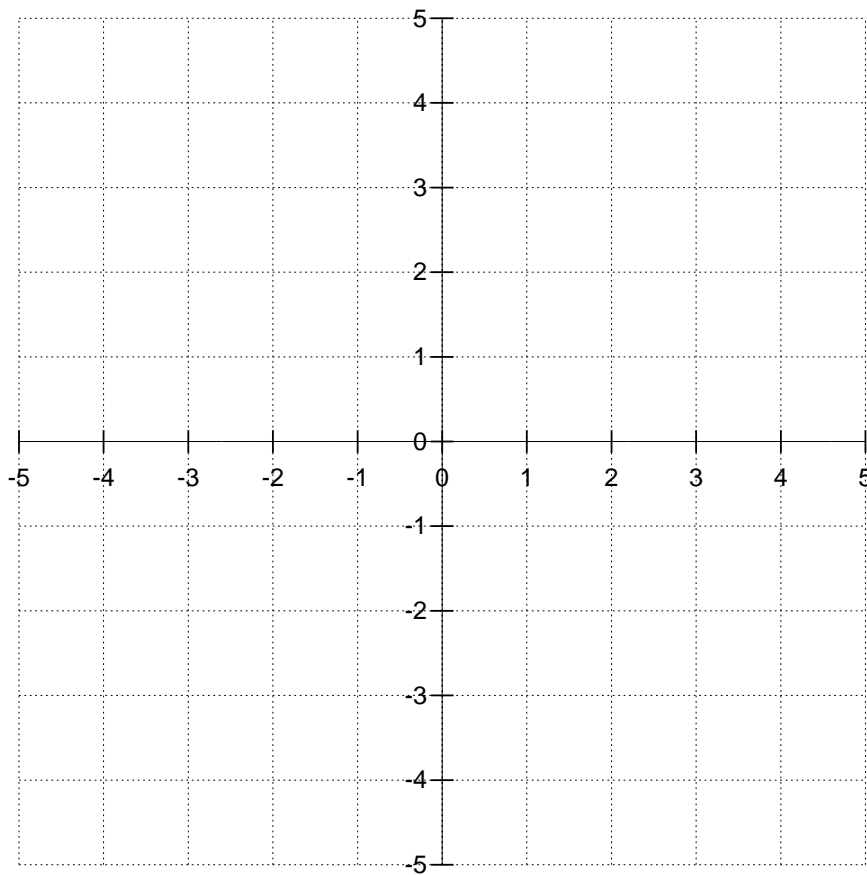
(c) Find all critical points of  $f$ , and determine whether each is a local minimum, a local maximum, or neither.

*(Note: This problem continues on the next page.)*

8. continued

(d) Find all inflection points.

(e) Sketch the graph  $y = f(x)$ .



9. (9 points) Find the linear approximation of the function

$$f(x) = \tan\left(\frac{\pi}{4} e^x\right)$$

at  $a = 0$  and use it to approximate  $f(0.2)$ .