Math 380 ***HOMEWORK #6*** ***due Tue, November 9***

Reading: Section 5.5 in [Zeitz].

Written assignment (4 problems).

W1. If $a, b, c > 0$, show that $(a+b+c)(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) \geq 9$.

W2. Prove the Arithmetic-Geometric Mean inequality for four numbers.

W3. Let $a_1 a_2 \ldots a_n = 1$ where $a_1, \ldots, a_n$ are positive real numbers. Show that

$$(1+a_1)(1+a_2)\ldots(1+a_n) \geq 2^n.$$

W4. Prove Bernoulli's inequality: $(1+x)^n \geq 1+nx$ if $n \geq 1, x \geq -1$.

Optional bonus problem.

Bonus 1. Let $a, b, c, d \geq 0$. Prove that $\frac{1}{a}+\frac{1}{b}+\frac{4}{c}+\frac{16}{d} \geq \frac{64}{a+b+c+d}$

Presentation assignment (4 problems).

P1. Let $n \geq 2$. Prove that $\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2} < 1$.

P2. Prove Arithmetic-Geometric Mean Inequality for three numbers. Hint: one way to do it uses W2.

P3. Let $a_1, \ldots, a_n, b_1, \ldots, b_n$ be real numbers. Prove that

$$\sqrt{a_1^2+b_1^2}+\sqrt{a_2^2+b_2^2}+\cdots+\sqrt{a_n^2+b_n^2} \geq \sqrt{(a_1+a_2+\cdots a_n)^2+(b_1+b_2+\cdots+b_n)^2}$$

P4. Let $a_1, \ldots, a_n$ be a sequence of positive numbers and let $b_1, \ldots, b_n$ be any permutation of the first sequence. Show that

$$\frac{a_1}{b_1}+\frac{a_2}{b_2}+\ldots+\frac{a_n}{b_n} \geq n$$