Reading Assignment: Read chapters 9.3, 9.4, 10.1, and 10.3 (10.1 and some of 10.3 were studied last quarter; presumably you can just skim through and recall the important results). Take a look at 10.2; we might get back to it when we reach Chapter 17.

Written Assignment: There are 5 mandatory problems in this assignment, one of which includes a "brownie points" question (which is optional).

Problem 1. (Problem 29 from Chapter 9) Find (list) all non-isomorphic simple graphs on four vertices.

Problem 2. Find a simple graph on 6 vertices with no non-trivial automorphisms (the trivial automorphism being the identity function). **Brownie points question:** prove that no such graph exists which has less than 6 vertices.

Problem 3. Let us revisit Problem 42 from Chapter 9.

Let G be a simple graph on vertex set $[n]$ in which each vertex has degree two, and let $g(n)$ be the number of such graphs; $g(0) = 1$, $g(1) = g(2) = 0$.

a) We have proved last time that $g(n)$ satisfies

$$g(n+3) = (n+2)g(n+2) + \binom{n+2}{2} g(n) \ .$$

Compute the exponential generating function $G(x)$ for $g(n)$.

b) Explain why is $G(x)$ different from $\sum_{n \geq 0} n! \frac{x^n}{n!} = \frac{1}{1-x}$, which is the exponential generating function for the numbers of n-permutations, even though n-permutations are also unions of disjoint cycles on the set $[n]$.

Problem 4. (Problem 23 from Chapter 10) Prove that in any tree T, all longest paths cross one another *at the same vertex*.

Problem 5. (Problem 26 from Chapter 10) Find (with proof!) the smallest tree with no non-trivial automorphisms (Hint: you may use Problem 2!)