Written Assignment: There are 3 mandatory problems in this homework.

Problem 1. Recall that the five regular convex polyhedra are the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron (see Table 1 below).

Show that the dual of a regular polyhedron is a regular polyhedron, then compute the dual of each of the five regular polyhedra. What do you notice?

Table 1: Regular polyhedra

Regular polyhedron	Edges per face	Vertex degree	Number of faces
Tetrahedron	3	3	4
Cube	4	3	6
Octahedron	3	4	8
Dodecahedron	5	3	12
Icosahedron	3	5	20

Problem 2. (Problem 18 from Chapter 12) Let G be a planar graph in which each face is either a 2-gon, a 3-gon, or a 4-gon (a 2-gon consists of two edges between a pair of vertices). Let p_2, p_3, p_4 respectively denote the number of faces which are 2-gons, 3-gons and 4-gons. Assume that each vertex in G has degree 4, and that $p_2 + p_3 = 8$.

Prove that $p_2 = 0$ and $p_3 = 8$.

Note: a planar graph of this type (or polyhedron, now that we know $p_2 = 0$) is called an *octahedrite*.

Problem 3.

(a) (Problem 12, Chapter 12) Is it true that if a connected graph satisfies $E \leq 3V - 6$, the graph is planar?

(b) (Problem 11, Chapter 12) All of the faces of a convex polyhedron are triangles or pentagons. Prove that the number of faces is even.