

Written Assignment: There are 3 mandatory problems in this homework.

Problem 1. This problem goes back to a problem on the midterm. We know that the K_6 has the property that, if colored with 2 colors, there always exists a monochromatic triangle. Similarly, if K_{17} is colored with 3 colors, there always exists a monochromatic triangle.

Show that this can be extended to form a sequence a_n such that, when K_{a_n} is colored with $n + 2$ colors, there always exists a monochromatic triangle, by finding a recurrence for a_{n+1} in terms of a_n . (Obviously, $a_0 = 6$ and $a_1 = 17$.)

Solve this recurrence to find a closed-form formula for a_n .

Problem 2. (Problem 20 from Chapter 13) Prove that if we color the edges of K_6 red or blue, then there will be at least *two* monochromatic triangles. (**Hint:** the procedure we used to show there is one monochromatic triangle says to pick a vertex; what if we look at more than one vertex?...)

Problem 3. (My own take on than Problem 19, Chapter 13). There are n guests at a house party, and the following facts are true:

- in any group of three people, there are two people who do not know each other, and
- in any group of seven people, there are two people who know each other.

Guests mingle and go from room to room. At some point, one of them looks around the living room and claims that she knows 20 of the people in the room, and has no idea who the other 9 are.

Prove that she is mistaken.