Math 462 **HOMEWORK #8** *due Friday, March 7*

Written Assignment: There are 5 mandatory problems in this assignment, and one "brownie points" problem.

Problem 1. (Problem 42 from Chapter 15) Find the average number of cycles of length 4 in a randomly selected graph on n vertices. (The probability of each edge being present is 1/2.)

Problem 2. (Problem 23 from Chapter 15) Prove that it is possible to 2-color the numbers from 1 to 1000 so that no monochromatic arithmetic progression of length 17 is formed. **Brownie points problem:** Generalize to n numbers. How big can you make k as a function of n such that there are still 2-colorings of 1 through n which have no monochromatic k-length arithmetic progressions?

Problem 3. A deck of cards has "regular" 52 cards and 2 jokers (54 cards total). Assume the deck is shuffled so that the cards are distributed randomly. Let p_k be the probability that, counting from the top of the deck, the first appearance of a joker happens on the kth card. Which k maximizes p_k? (**Hint:** compute p_k for each k, then compare p_k and p_{k+1}.)

Problem 4. What is the average number of cycles in a randomly selected permutation?

Problem 5. Assume that we have $(4n+2)$ disks; $2n+1$ have one face painted red, and one painted blue; n of them have both faces painted red, and $n+1$ have both faces painted blue. One disk is selected by an opponent and placed on a table, and you have to guess the color of the face not visible.

- Assume you don't get to see the selected disk, and can only say "same as the visible one" or "different from the visible one". What should you say to maximize your chances of getting it right?
- Assume you do see the color on the top. What should you say now?

Note: this is a fun problem with a counterintuitive answer. =)