Math 125, Section G, Spring 2008, Solutions to Midterm I

1. Evaluate the following integrals. (5+5 points)

   (a) $\int_0^2 |t^2 - 4t + 3| dt$

   $$t^2 - 4t + 3 = (t-1)(t-3)$$

   So

   $$\int_0^2 |t^2 - 4t + 3| dt = \int_0^1 t^2 - 4t + 3) dt - \int_1^2 (t^2 - 4t + 3) dt$$

   $$= (\frac{t^3}{3} - 2t^2 + 3t)|_0^1 - (\frac{t^3}{3} - 2t^2 + 3t)|_1^2 = 2$$

   (b) $\int_0^2 \frac{x^2}{1+x^6} dx$ Using substitution

   $$u = x^3 \qquad du = 3x^2 dx$$

   $$\frac{1}{3}\int_0^8 \frac{1}{1+u^2} du = \frac{1}{3}(\arctan(8) - \arctan(0)) = \frac{\arctan(8)}{3}$$

2. Determine whether the statement is True or False. If it is true, explain why. If it is False, explain why or give and example that disproves the statement. (1 point for the answer, 1 point for the explanation for each question.)

   (a) $\int_{-1}^{1} \left(2x^3 - \sin x + \frac{x\cos x}{1+x^2}\right) dx = 0$
   TRUE. The function is odd.

   (b) $\frac{d}{dx}\int_{2}^{x^3} \tan(t)dt = \tan(x^3)$
   FALSE. Using FTC and the Chain Rule

$$\frac{d}{dx}\int_{2}^{x^3} \tan(t)dt = 3x^2\tan(x^3)$$

   (c) If f is continuous on $[a, b]$ then $\int_a^b xf(x)dx = x\int_a^b f(x)dx$
   FALSE. Left hand side is a number, right hand side is a function!

   (d) All continuous functions have antiderivatives.
   TRUE. This is the FTC. An antiderivative for $f(x)$ is

$$\int_0^x f(t)dt$$

   (e) All continuous functions have derivatives.
   FALSE. $|x|$ is continuous but its derivative is not defined at $x = 0$.

3. A region is bounded below by $y = x^2$, above by $y = 4$ and to the left by $y = 4x$.

(a) Sketch the region.(1 point)

(b) Set up an integral to calculate the area of the region with respect to x.(3 points)

$$\int_0^1 (4x - x^2)dx + \int_1^2 (4 - x^2)dx$$

(c) Set up an integral to calculate the area of the region with respect to y.(3 points)

$$\int_0^4 (\sqrt{(}y) - y/4)dy$$

(d) Choose one of your integrals above to evaluate and find the volume. (3 points)

$$\int_0^1 (4x - x^2)dx + \int_1^2 (4 - x^2)dx = (2x^2 - x^3/3)|_0^1 (4x - \frac{x^3}{3})|_1^2 = \frac{10}{3}$$

OR

$$\int_0^4 (\sqrt{(}y) - y/4)dy = (\frac{2}{3}y^{3/2} - \frac{y^2}{8})|_0^4 = \frac{10}{3}$$

4. A hole of radius 3 cm is bored through the center of a sphere of radius 5 cm. Find the volume of the remaining part of the sphere.

   The region to the right of $x = 3$ and inside the circle $x^2 + y^2 = 25$ is revolved around the y-axis.

   The volume calculated using washers/disks:

$$\int_{-4}^{4} \pi((25 - y^2) - 9)dy = 2\pi \int_{0}^{4} (16 - y^2)dy = 2\pi(16y - \frac{y^3}{3})|_0^4 = \frac{256\pi}{3}$$

   The volume calculated using cylindrical shells:

$$\int_{3}^{5} 2\pi x(2\sqrt{25 - x^2})dx = 2\pi \int_{16}^{0} -\sqrt{u}du = \frac{4\pi}{3}u^{3/2}|_0^{16} = \frac{256\pi}{3}$$