Math 125, Sections E and F, Fall 2011, Midterm I

October 20, 2011

Name_____

TA/Section_____

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**
- You may use a calculator which does not graph and which is not programmable. Even if you have a calculator, give me exact answers. ($\frac{2\ln 3}{\pi}$ is exact, 0.7 is an approximation for the same number.)
- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please $\boxed{\text{BOX}}$ your final answer.

Question	points
1	
2	
3	
4	
Total	

1. Evaluate the following integrals.

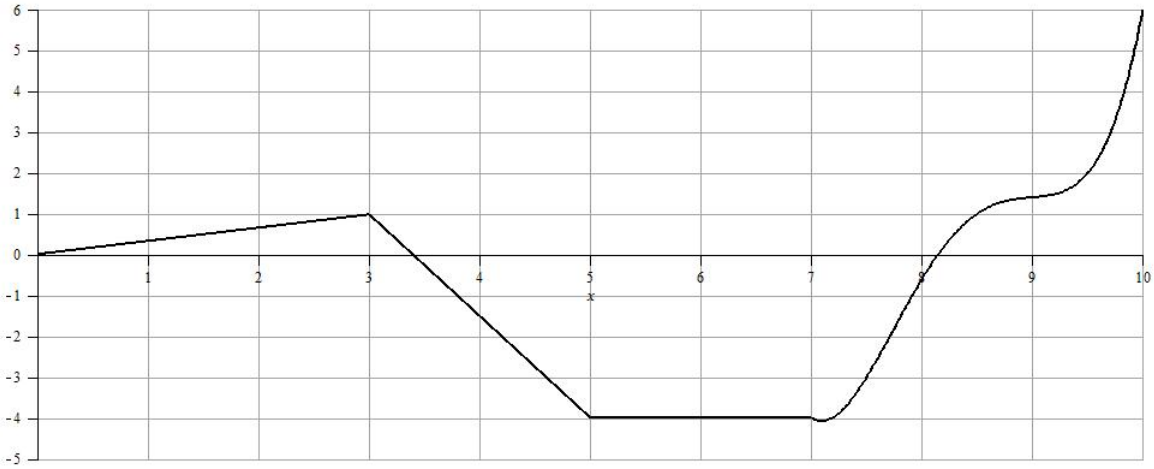
(a) (1 point) $\displaystyle\int_{-1}^{1} \frac{\sin t}{1+t^2} \; dt$

(b) (2 points) $\displaystyle\int \frac{\sqrt{x} - 5x^2}{x} \; dx$

(c) (3 points) $\displaystyle\int_{0}^{1} (6 - 4x)^5 \; dx$

(d) (4 points) $\displaystyle\int_{0}^{3} |x^2 - 4| \; dx$

2. Let g be the function whose graph is given below. The pieces from $x = 0$ to $x = 7$ are lines.



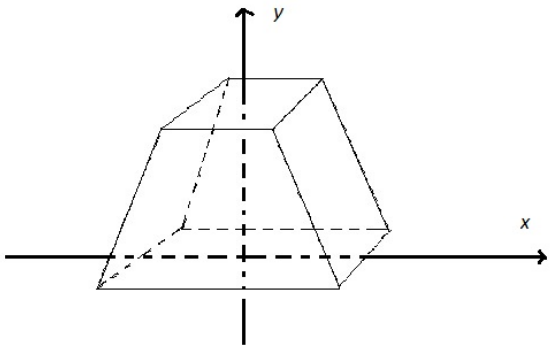
(a) (5 points) Compute $\int_6^3 g(x)dx$. Give me an exact answer.

(b) (3 points) Let $f(x) = \int_2^{x^2} g(t)dt$. Compute $f'(5/2)$.

(c) (2 points) Approximate $\int_6^{10} g(x)dx$ with $n = 4$ and using midpoints.

3. Find the volume of a frustum of a pyramid with square base of side 6 and a square top of side 4 and height 12 by completing the following steps.

   (a) (1 point) Slice the pyramid horizontally. What are the cross sections?



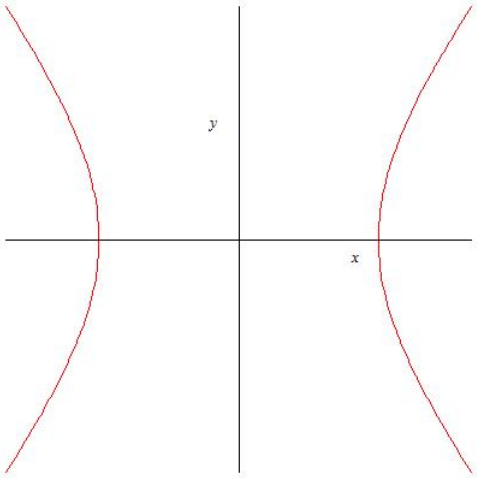
   (b) (5 points) Find the volume ΔV of a cross section in terms of y. Your answer should have a Δy in it. Hint: Draw a trapezoid cross section picture on the xy-plane to help you.

   (c) (1 point) Use your answer from above to write an integral which represents the volume.

   (d) (3 points) Evaluate the integral to find the volume.

4. Let R be the region in the first quadrant bounded on the right by the hyperbola $x^2 - y^2 = 8$, on the left by $y = \frac{x}{3}$ and below by the x-axis. A sketch of the hyperbola is provided below.

   (a) (2 points) Sketch the region showing all relevant points of intersection.



   (b) (3 points) Set up and integral to compute the area of the region. Do not evaluate the integral.

   (c) (5 points) Set up an integral to find the volume of the solid obtained by rotating the region R about the line $x = 7$. Do not evaluate the integral.