Math 125, Sections E and F, Fall 2011, Solutions to Midterm I

1. Evaluate the following integrals.

   (a) (1 point) $\int_{-1}^{1} \frac{\sin t}{1+t^2} \quad dt = 0$ becuase the integrand is an odd function.

   (b) (2 points) $\int \frac{\sqrt{x} - 5x^2}{x} \quad dx = 2\sqrt{x} - 2.5x^2 + C$

   (c) (3 points) Let $u = 6 - 4x$, then

   $$\int_0^1 (6-4x)^5 \quad dx = -\frac{1}{4}\int_6^2 u^5 du = \frac{5824}{3}.$$

   (d) (4 points)

   $$\int_0^3 |x^2 - 4| \quad dx = \int_0^2 -(x^2-4) \quad dx + \int_2^3 (x^2-4) \quad dx = \frac{23}{3}.$$

2. Let g be the function whose graph is given below. The pieces from $x = 0$ to $x = 7$ are lines.

   (a) (5 points) The equation of the line from $x = 3$ to $x = 5$ is $y - 1 = \frac{5}{2}(x-3)$ so the line crosses the x axis at $x = 17/5$. Therefore,

   $$\int_6^3 g(x)dx = -\int_3^6 g(x)dx = -\left[\frac{1}{5} - \frac{16}{5} - 4\right] = 7.$$

   (b) (3 points) Let $f(x) = \int_2^{x^2} g(t)dt$. Compute $f'(5/2)$.
   By the Fundamental Theorem of Calculus and the chain rule

   $$f'(x) = g(x^2) \cdot 2x$$

   so

   $$f'(5/2) = g(25/4) \cdot 5 = -4 \cdot 5 = -20.$$

   (c) (2 points) Approximate $\int_6^{10} g(x)dx$ with $n = 4$ and using midpoints. Here $\Delta x = 1$ so

   $$\int_6^{10} g(x)dx \approx g(6.5) + g(7.5) + g(8.5) + g(9.5) = -4 - 3 + 1 + 2 = -4.$$

3. Find the volume of a frustum of a pyramid with square base of side 6 and a square top of side 4 and height 12 by completing the following steps.

   (a) (1 point) Slice the pyramid horizontally. What are the cross sections? Squares

   (b) (5 points) Find the volume ΔV of a cross section in terms of y. Your answer should have a Δy in it. Hint: Draw a trapezoid cross section picture on the xy-plane to help you.

$$\Delta V \approx \left[2\left(-\frac{1}{12}y+3\right)\right]^2 \Delta y$$

   (c) (1 point) Use your answer from above to write an integral which represents the volume.

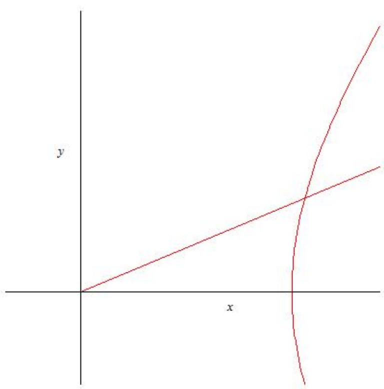
$$V = \int_0^{12} \left[2\left(-\frac{1}{12}y+3\right)\right]^2 dy.$$

   (d) (3 points) Evaluate the integral to find the volume. Let $u = -\frac{1}{12}y+3$, then

$$V = \int_0^{12} \left[2\left(-\frac{1}{12}y+3\right)\right]^2 dy = -48\int_3^2 u^2 du = 304.$$

4. Let R be the region in the first quadrant bounded on the right by the hyperbola $x^2 - y^2 = 8$, on the left by $y = \frac{x}{3}$ and below by the x-axis. A sketch of the hyperbola is provided below.

   (a) (2 points) Sketch the region showing all relevant points of intersection.



   The three corners of the triangular region are $(0,0), (3,1), (2\sqrt{2}, 0)$.

   (b) (3 points) Set up and integral to compute the area of the region. Do not evaluate the integral.

$$\int_0^1 \sqrt{y^2+8} - 3y \quad dy$$

   (c) (5 points) Set up an integral to find the volume of the solid obtained by rotating the region R about the line $x = 7$. Do not evaluate the integral.

$$\pi \int_0^1 \left[(7-3y)^2 - \left(7-\sqrt{y^2+8}\right)^2\right] dy$$