Math 125, Sections F and G, Autumn 2012, Midterm II

November 15, 2012

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Instructions.

- There are 4 questions. The exam is out of 40 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**

- You may use a calculator which does not graph and which is not programmable. Even if you have a calculator, give me exact answers. ($\frac{2\ln 3}{\pi}$ is exact, 0.7 is an approximation for the same number.)

- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please ⎡BOX⎤ your final answer.

- You can use any of the integrals on page 495 of your textbook wthithout any explanation. Anything else must be justified by your work.

Question	points
1	
2	
3	
4	
Total	

1. Evaluate the following integrals.

   (a) (5 points) $\int x \ln(x+4) \; dx$

   (b) (5 points) $\int_{1/2}^{1} \dfrac{dx}{x^3\sqrt{4x^2-1}}$

2. For the improper integral
$$\int_1^\infty \frac{12x + 6}{x^3 + 5x^2 + 6x} dx$$

(a) (2 points) Determine if it converges or diverges using the comparison theorem. Give a brief explanation.

(b) (8 points) Now evaluate it to see if it diverges or converges. If it converges, what value does it converge to?

3. This question concerns the length of the curve $y = \left(\frac{2}{3}x\right)^{\frac{3}{2}}$ from the point $(3/2, 1)$ to the point $(6, 8)$.

(a) (4 points) Set up two integrals for the length of the curve. One should end in dx, the other in dy.

(b) (3 points) Pick one of the integrals and evaluate it to find the exact length of the curve.

(c) (2 points) Approximate the OTHER integral using Simpson's rule with $n = 6$.

(d) (1 point) Find the percentage error you made in your approximation in part (c) by comparing with the exact value you found in part (b).

$$\text{Percentage Error} = \frac{\text{actual value} - \text{approximate value}}{\text{actual value}} \times 100\%$$

4. A tank is formed by rotating the parabola $y = \frac{1}{5}x^2$ between $x = 0$ and $x = 5$ about the y-axis. The units are in meters. Initially, it is full of water of density 1000 kg/m^3. It is shown partially full in the picture below. The depth of the water in the tank is measured at its deepest point with the meter stick show in the picture.
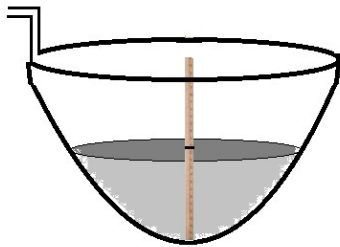
(a) (8 points) Water is pumped out from an outlet 2 meters above its top until the depth of the remaining water is 1 meter. Find the work done.

(b) (2 points) Find the weight of the water left in the tank.