## Math 125, Sections F and G, Autumn 2012, Solutions to Midterm I

1. Evaluate the following integrals.

   (a) $\int_1^4 \frac{9x^3+\sqrt{x}}{x} \quad dx = \int_1^4 9x^2 + x^{-1/2} dx = 3x^3 + 2\sqrt{x}\Big|_1^4 = 191$

   (b) $\int_1^e \frac{\ln x}{x} \quad dx$
   Let $u = \ln x$ then $du = \frac{1}{x}dx$ so
   $$\int_1^e \frac{\ln x}{x} \quad dx = \int_0^1 u \quad du = \frac{1}{2}.$$

   (c) (3 points) $\int \frac{x^2+2x+3}{x-1} \quad dx$
   Let $u = x-1$ then $du = dx$ so
   $$\int \frac{x^2+2x+3}{x-1} \quad dx = \int \frac{(u+1)^2+2(u+1)+3}{u} \quad du$$
   $$= \int u + 4 + \frac{6}{u} \quad du = \frac{1}{2}u^2 + 4u + 6\ln|u| + C = \frac{1}{2}(x-1)^2 + 4(x-1) + 6\ln|x-1| + C$$

2. Let
   $$f(x) = \int_1^{x^2+1} e^{\sin t} dt$$

   Answer the following questions about $f(x)$.

   (a) $f(0) = \int_1^1 e^{\sin t} dt = 0$

   (b) $f'(x) = 2xe^{\sin(x^2+1)}$

   (c) Is $f(-2) > f(1)$?
   $$f(-2) - f(-1) = \int_1^5 e^{\sin t} dt - \int_1^2 e^{\sin t} dt = \int_2^5 e^{\sin t} dt > 0$$
   since $e^{\sin t} > 0$ and $2 < 5$. So, yes, $f(-2) > f(1)$.

   (d) Is $f$ increasing or decreasing at $x = 2$?
   From part (b), $f'(2) = 2 \cdot 2 \cdot e^{\sin 5} > 0$ so $f$ is increasing at $x = 2$.

   (e) Approximate $f(1)$ with $n = 4$ and using right points. Round your answer to three decimal points.
   We have
   $$f(1) = \int_1^2 e^{\sin t} dt$$
   so $\Delta t = 1/4$ and
   $$f(1) \approx \frac{1}{4}\left[e^{\sin(5/4)} + e^{\sin(6/4)} + e^{\sin(7/4)} + e^{\sin(8/4)}\right] \approx 2.613$$

3. The following questions are about the region whose graph is given below. The intersection points are at $(1, 10)$, $(2, 8)$ and $(0, 0)$. The curve has the equation $y = x^3$.

   (a) Set up integral(s) ending in $dx$ to find the area.

$$A = \int_0^1 10x - x^3 dx + \int_1^2 (-2x + 12) - x^3 dx$$

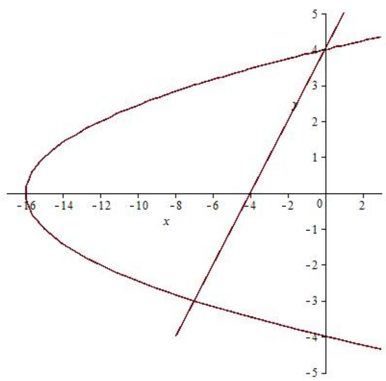
   (b) Set up integral(s) ending in $dy$ to find the area.

$$A = \int_0^8 y^{1/3} - \frac{y}{10} dy + \int_8^{10} \left(\frac{-y}{2} + 6\right) - \frac{y}{10} dy$$

   (c) Evaluate your answer in part (a) or (b) to find the area. $A = 10$.

4. The following questions are about the region between the line $y = x + 4$ and the parabola $x = y^2 - 16$.

   (a) Sketch the region labeling all intersection points.
   The intersection points are $(-7, -3)$ and $(0, 4)$.



   (b) Set up an integral to find the area of the region. Do not integrate.

$$A = \int_{-3}^{4} (y - 4) - (y^2 - 16) dy$$

   (c) Set up an integral to find the volume generated by rotating the region about the $y$-axis. Do not integrate.

$$V = \int_{-3}^{4} \pi \left[ \left(0 - (y^2 - 6)\right)^2 - \left(0 - (y - 4)\right)^2 \right] dy$$

   (d) Set up an integral to find the volume generated by rotating the region about the horizontal line $y = -6$. Do not integrate.

$$V = \int_{-3}^{4} 2\pi(y + 6) \left[ (y - 4) - (y^2 - 16) \right] dy$$