Math 125, Section E, Spring 2011, Solutions to Midterm I

1. Evaluate the following integrals.

   (a) (4 points)

   $$\int_0^{1/2} t\sec^2(t^2) \quad dt$$

   After the substitution

   $$u = t^2 \qquad\qquad du = 2tdt$$

   the integral becomes

   $$\int_0^{1/4} \frac{1}{2}\sec^2 u \quad du = \frac{1}{2}\tan\left(\frac{1}{4}\right)$$

   (b) (4 points)

   $$\int (e^x + e^{-x})^2 \quad dx = \int e^{2x} + 2 + e^{-2x} dx = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$$

   (c) (4 points)

   $$\int_0^5 x\sqrt{x+4} \quad dx$$

   After the substitution

   $$u = x + 4 \quad du = dx$$

   the integral becomes

   $$\int_4^9 (u-4)\sqrt{u} \quad du = \int_4^9 u^{3/2} - 4u^{1/2} du = \frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2}\Big|_4^9 = \frac{506}{15}$$

2. (10 points) Define $g(x) = \int_5^x f(t)dt$ where f is the function whose graph is shown below. All the critical points of the graph have integer coordinates.

   (a) $g(6) = 3$, (b) $g(0) = -9$, (c) $g'(8) = -3$, (d) $g'(1) = 1$

   (e) $g''(2) = f'(2)$ is not defined, (f) $g''(3) = f'(3) = 1$

   (g) Let $h(x) = \int_x^{x^2} f(t)dt$. What is $h'(2)$?

   $$h'(x) = -f(x) + 2xf(x^2) \text{ so } h'(2) = 11.$$

   (h) $\int_0^2 g(x)dx = \int_0^2 (x-9)dx = -16.$

3. An object is moving along the x-axis with acceleration at time $t \geq 0$ given by

$$a(t) = -\frac{60}{(t+3)^2}\,\text{ft/sec}^2.$$

The object has initial velocity $v(0) = 5$ ft/sec.

(a) At what time does the object reverse direction?

$$v(t) = \frac{60}{t+3} - 15$$

so the object reverses direction at $t = 1$.

(b) What is the total distance travelled by the object from $t = 0$ to $t = 4$ seconds?

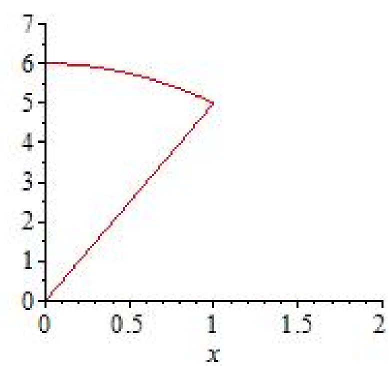
$$\text{distance} = \int_0^4 |v(t)|dt = \int_0^1 \frac{60}{t+3} - 15 \quad dt - \int_1^4 \frac{60}{t+3} - 15 \quad dt$$

$$= \left(60\ln(t+3) - 15t\Big|_0^1\right) - \left(60\ln(t+3) - 15t\Big|_1^4\right)$$

$$= 60\ln\left(\frac{16}{21}\right) + 30 \text{ ft.}$$

4. Let R be the region bounded above by the curve $y = -x^2 + 6$, on the right by $y = 5x$ and on the left by the y-axis.

(a) Sketch the region showing all relevant points of intersection.



(b) Find the volume of the solid obtained by rotating the region R about the line $y = 7$.

$$\text{Volume} = \int_0^1 \pi\left[(7-5x)^2 - (7-(-x^2+6))^2\right]dx = \int_0^1 \pi\left[(7-5x)^2 - (1+x^2)^2\right]dx$$

$$= \pi\frac{(7-5x)^3}{3(-5)}\Big|_0^1 - \pi\int_0^1 1 + 2x^2 + x^4 dx$$

$$= \pi\left(-\frac{2^3}{15} + \frac{7^3}{15}\right) - \pi\left(x + \frac{2}{3}x^3 + \frac{x^5}{5}\Big|_0^1\right) = \frac{307}{15}\pi$$