Math 125, Section A, Spring 2012, Solutions to Midterm I

1. Evaluate the following integrals.

   (a) $\int_0^3 \frac{t^2+t-1}{t+2}\, dt$. Do the substitution $u = t+2$ to get

   $$\int_2^5 \frac{(u-2)^2+(u-2)-1}{u} du = \int_2^5 u-3+\frac{1}{u} du = \frac{u^2}{2} - 3u + \ln|u|\Big|_2^5 = \frac{3}{2} + \ln\left(\frac{5}{2}\right).$$

   (b) $\int \frac{\arctan x}{x^2+1}\, dx$. Do the substituion $u = \arctan x$ to get

   $$\int u \quad du = \frac{u^2}{2} + C = \frac{(\arctan x)^2}{2} + C.$$

   (c) $\int_0^1 7x^2 + e^x - \sin x \quad dx = \frac{7x^3}{3} + e^x + \cos x\Big|_0^1 = \frac{1}{3} + e + \cos 1.$

2. Let

   $$g(x) = \int_x^{x^2+1} \ln\left(t^3+1\right) dt$$

   (a) Compute $g'(2)$.
   By the Fudamental Theorem of Calculus and the Chain Rule

   $$g'(x) = \ln\left(\left(x^2+1\right)^3+1\right)(2x) - \ln\left(x^3+1\right)$$

   so

   $$g'(2) = \ln\left(\left(2^2+1\right)^3+1\right)(2\cdot 2) - \ln\left(2^3+1\right) = 4\ln 126 - \ln 9.$$

   (b) Approximate $g(2)$ with $n = 6$ and using left points. Is your approximations more than or less than the actual value of $g(2)$?

   $$g(2) = \int_2^5 \ln\left(t^3+1\right) dt$$

   $\Delta x = \frac{5-2}{6} = \frac{1}{2}$so

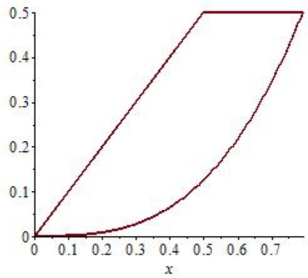
   $$L_6 = \left[\ln\left(2^3+1\right) + \ln\left(\left(\frac{5}{2}\right)^3+1\right) + \ln\left(3^3+1\right) + \ln\left(\left(\frac{7}{2}\right)^3+1\right) + \ln\left(4^3+1\right) + \ln\left(\left(\frac{9}{2}\right)^3+1\right)\right]\cdot\frac{1}{2}$$

   $$= \left[\ln 9 + \ln\frac{133}{8} + \ln 28 + \ln\frac{351}{8} + \ln 65 + \ln\frac{737}{8}\right]\cdot\frac{1}{2}$$

   $$= [\ln 9 + \ln 133 + \ln 28 + \ln 351 + \ln 65 + \ln 737 - 3\ln 8]\cdot\frac{1}{2} \approx 10.41$$

3. To find the area of the region which is bounded by $y = x^3$ on the right, $y = x$ on the left and the horizontal line $y = 0.5$ at the top,

   (a) Sketch the region labelling all necessary intersection points.



   (b) Set up integral(s) ending in dx to find the area.

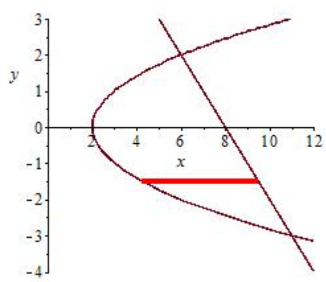
$$\int_0^{\frac{1}{2}} x - x^3 \quad dx + \int_{\frac{1}{2}}^{2^{-\frac{1}{3}}} \frac{1}{2} - x^3 \quad dx$$

   (c) Set up integral(s) ending in dy to find the area.

$$\int_0^{\frac{1}{2}} \sqrt[3]{y} = y \quad dy = \frac{4y^{4/3}}{3} - \frac{y^2}{2}\Big|_0^{\frac{1}{2}} = \frac{2}{\sqrt[3]{2}} - \frac{1}{8}$$

   (d) Evaluate your answer in part (b) or (c) to find the area. (Or do both to chech your work.)

4. Find the volume of the solid obtained by rotating the region between $x = y^2 + 2$ and $x + y = 8$ about the y-axis.



The intersection points are given by the solutions to

$$y^2 + 2 = 8 = 6$$

so $y = -3$ and $y = 2$. Since the parabola equation is of the form $x = f(y)$ we integrate with respect to y which means we need a horizontal Δy strip as shown in the picture. The rotation about the y- axis gives a disk with a hole. Therefore, the volume is given by

$$\pi \int_{-3}^{2} (8-y)^2 - (y^2+1)^2 dy = \pi \int_{-3}^{2} 63 - 16y - y^2 - y^4 \quad dy$$

$$= 63y - 8y^2 - \frac{y^3}{3} - \frac{y^5}{5}\Big|_{-3}^{2} = \left(126 - 32 - \frac{8}{3} - \frac{32}{5}\right) - \left(-189 - 72 + 9 + \frac{243}{5}\right) = \frac{961}{3}.$$