Math 125, Sections E and F, Midterm I

April 25, 2013

Name Key

TA/Section

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**
- You may use a calculator which does not graph and which is not programmable. Even if you have a calculator, give me exact answers. ($\frac{2\ln 3}{\pi}$ is exact, 0.7 is an approximation for the same number.)
- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please $\boxed{\text{BOX}}$ your final answer.

Question	points
1	
2	
3	
4	
Total	

1. Evaluate the following integrals.

(a) (3 points) $\int_0^1 5x^3 + \sqrt[3]{x} - \frac{1}{1+x^2} \quad dx$

$$= \frac{5x^4}{4} + \frac{3}{4}x^{4/3} - \arctan x \Big|_0^1$$

$$= \left(\frac{5}{4} + \frac{3}{4} - \arctan 1\right) - (0)$$

$$= 2 - \frac{\pi}{4}$$

(b) (3 points) $\int_1^e \frac{\sqrt{\ln x + 3}}{x} \quad dx$

$u = \ln x + 3$

$du = \frac{1}{x} dx$

$x = 1 \rightarrow u = \ln 1 + 3 = 3$

$x = e \rightarrow u = \ln e + 3 = 4$

$$= \int_3^4 \sqrt{u}\, du$$

$$= \frac{2}{3} u^{3/2} \Big|_3^4$$

$$= \frac{2}{3}\left(4^{3/2} - 3^{3/2}\right)$$

$$= \frac{2}{3}\left(8 - 3\sqrt{3}\right)$$

(c) (3 points) $\int \frac{x}{2x+3} \quad dx$

$u = 2x + 3 \longrightarrow \frac{u-3}{2} = x$

$du = 2x\, dx \longrightarrow \frac{du}{2} = x\, dx$

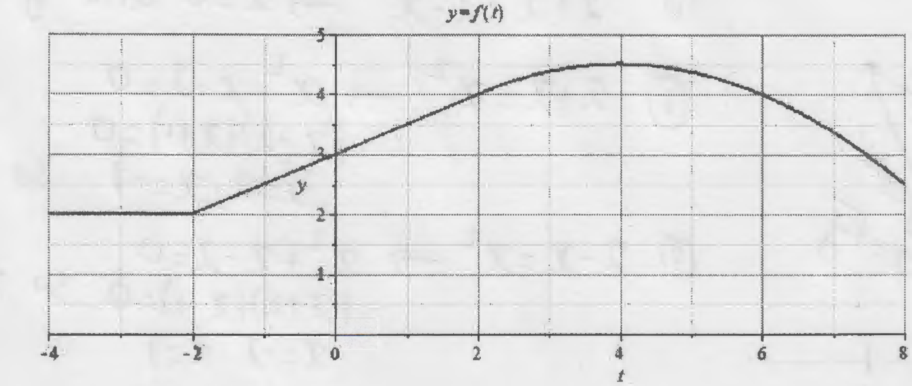
$$= \int \frac{\frac{u-3}{2}}{u} \frac{du}{2}$$

$$= \frac{1}{4} \int \frac{u-3}{u}\, du$$

$$= \frac{1}{4} \int 1 - \frac{3}{u}\, du = \frac{1}{4}\left(u - 3\ln|u|\right) + C$$

$$= \frac{1}{4}\left(2x + 3 - 3\ln|2x+3|\right) + C$$

2. The following is the graph of $y = f(t)$.



Define $g(x) = \int_0^x f(t)dt$ and $h(x) = \int_1^{x^2} f(t)dt$. Answer the following questions.

(a) (2 points) $g(-2) =$

$$\int_0^{-2} f(t)dt = -\int_{-2}^{0} f(t)dt = \left(\text{area under } f \text{ between } -2 \text{ and } 0\right)\cdot(-1) = -1\left(2\cdot 2 + \frac{2\cdot 1}{2}\right) = -5$$

(b) (2 points) $g'(4) =$

$g'(x) = f(x)$ by FTC

so $g'(4) = f(4) = 4.5$ or $\frac{9}{2}$

(c) (2 points) $h'(-1) =$

$h(x) = g(x^2)$ so $h'(x) = g'(x^2)\cdot 2x = f(x^2)(2x)$

$h'(-1) = f\left((-1)^2\right)\cdot 2(-1) = -2f(1) = -2(3.5) = -7$

(d) (2 points) Estimate $g(8)$ using $n = 4$ and midpoints.

$g(8) \approx [f(1)+f(3)+f(5)+f(7)]\cdot\left(\frac{8-0}{4}\right)$

$\approx (3.5+4.3+4.3+3.3)\,2 = 30.8$

(e) (2 points) $\int_{-2}^{0} g(x)dx$.

on $[-2,0]$, $g'(x) = f(x) = \frac{1}{2}x + 3$

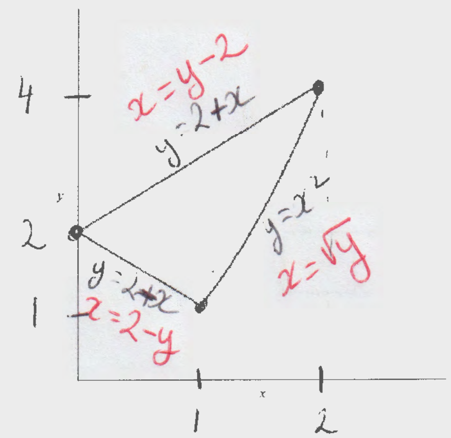
so $g(x) = \frac{-x^2}{2} + 3x + C$ since $g(0) = \int_0^0 f(t)dt = 0$, $C = 0$

so $g(x) = -\frac{x^2}{2} + 3x$

$$\int_{-2}^{0} \frac{-x^2}{2} + 3x\, dx = -\frac{x^3}{6} + \frac{3x^2}{2}\Big|_{-2}^{0}$$

$$= 0 - \left(\frac{8}{6} + 6\right) = \frac{-44}{6} = \frac{-22}{3}$$

3. The region shown below is bounded by $y = x^2$, $y = 2 - x$ and $y = 2 + x$.



① $2+x=2-x \rightarrow x=0$ and $y=2$

② $2+x=x^2 \rightarrow x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2,\ x=-1$ so $x=2, y=4$

③ $2-x=x^2 \rightarrow x^2+x-2=0$
$(x+2)(x-1)=0$
$x=-2 \quad x=1$ so $x=1 \quad y=1$

(a) (3 points) Label the functions and the points of intersection on the graph.

(b) (3 points) Set up integral(s) ending in dx to find the area.

$$A = \int_0^1 2+x-(2-x)\,dx + \int_1^2 2+x-x^2\,dx$$

(c) (3 points) Set up integral(s) ending in dy to find the area.

$$A = \int_1^2 \sqrt{y}-(2-y)\,dy + \int_2^4 \sqrt{y}-(y-2)\,dy$$

(d) (2 points) Evaluate your answer in part (a) or (b) to find the area. (Or do both and check your answer.)

$$(b) = \int_0^1 2x\,dx + \int_1^2 2+x-x^2\,dx = x^2\Big|_0^1 + 2x+\frac{x^2}{2}-\frac{x^3}{3}\Big|_1^2$$

$$= \cancel{1}-0 + \left(\cancel{4}+\cancel{2}-\frac{8}{3}\right)-\left(\cancel{2}+\frac{1}{2}-\frac{1}{3}\right) = 5-\frac{7}{3}-\frac{1}{2} = \frac{30-14-3}{6} = \frac{13}{6}$$

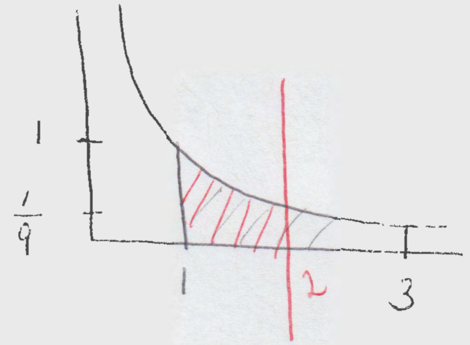
$$(c) = \frac{2}{3}y^{3/2}-2y+\frac{y^2}{2}\Big|_1^2 + \frac{2}{3}y^{3/2}-\frac{y^2}{2}+2y\Big|_2^4$$

$$= \left(\cancel{\frac{2}{3}2^{3/2}}-4+\cancel{2}\right)-\left(\frac{2}{3}-\cancel{2}+\frac{1}{2}\right)+\left(\frac{2}{3}\cdot 8-\cancel{8}+\cancel{8}\right)-\left(\cancel{\frac{2}{3}2^{3/2}}-2+4\right)$$

$$= -2+\frac{14}{3}-\frac{1}{2} = \frac{-12+28-3}{6} = \frac{13}{6}$$

4. (10 points) The region R is above the x-axis, below the curve $y = 1/x^2$ and between the vertical lines $x = 1$ and $x = 3$. Answer the following questions.

(a) (1 point) Sketch the region labeling all necessary points of intersection.



(b) (2 points) Set up an integral to compute the volume of the solid obtained by rotating the region about the x-axis. Do NOT integrate.

$$\int_1^3 \pi \left(\frac{1}{x^2}\right)^2 dx$$

(c) (2 points) Set up an integral to compute the volume of the solid obtained by rotating the region about the y-axis. Do NOT integrate.

$$\int_1^3 2\pi x \cdot \frac{1}{x^2}\, dx$$

(d) (3 points) Set up an integral to compute the volume of the solid obtained by rotating the region about the horizontal line $y = 4$. Do NOT integrate.

$$\int_1^3 \pi \left[(4-0)^2 - \left(4 - \frac{1}{x^2}\right)^2 \right] dx$$

(e) (2 points) Set up an integral to compute the volume of the solid obtained by rotating the region about the vertical line $x = 2$. Do NOT integrate.

$$\int_1^2 2\pi (2-x) \frac{1}{x^2}\, dx$$