Math 125, Section I, Winter 2011,Solutions to Midterm I

1. Evaluate the following integrals.

   (a)
$$\int_0^3 t^3\sqrt{1+t^4}dt$$
$$u = 1 + t^4 du = 4t^3 dt$$
$$\int_0^3 t^3\sqrt{1+t^4}dt = \int_1^{82} \frac{1}{4}\sqrt{u}du = \frac{1}{6}u^{3/2}\Big|_1^{82} = \frac{82^{3/2}-1}{6}$$

   (b) Here the substitutions $u = x + 1$ or $u = \sqrt{x+1}$ both work. If $u = x + 1$, the $du = dx$ so
$$\int \frac{x+3}{\sqrt{x+1}}dx = \int \frac{u+2}{\sqrt{u}}du = \frac{2}{3}u^{3/2} + 4\sqrt{u} + C = \frac{2}{3}(x+1)^{3/2} + 4\sqrt{x+1} + C$$

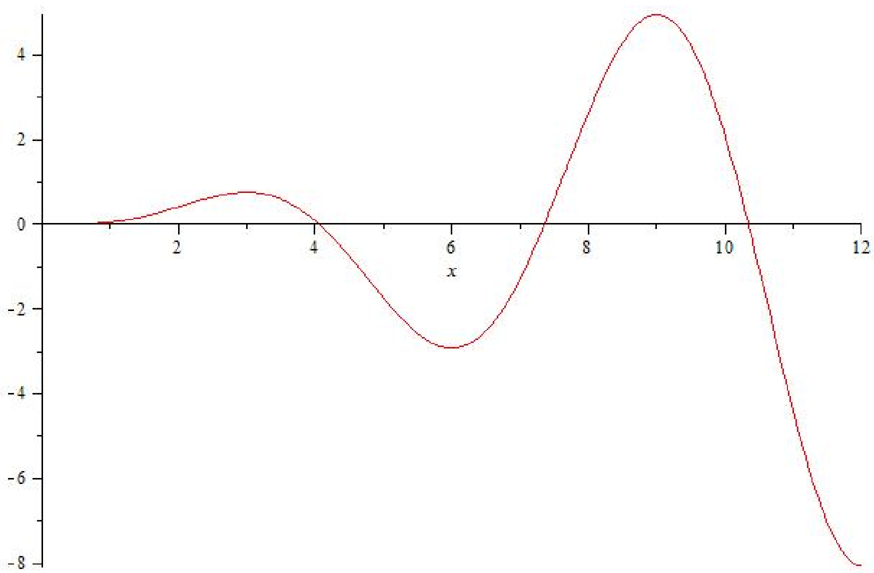
2. Define $g(x) = \int_0^x f(t)dt$ where f is the function whose graph is shown below.

   (a) At what values of x does g have a local maximum? $3, 9$

   (b) At what values of x does g have a local minimum? $6, 12$

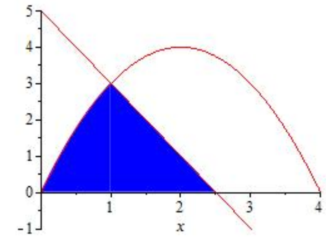
   (c) On what intervals is g concave up? The endpoints can be approximate here. Since $g''(x) = f'(x)$, the graph of g is concave up when $f' > 0$ so when f is increasing. The intervals approximately are $[0, 2.2]$, $[4.8, 7.8]$ and $[10.5, 12]$.

   (d) Sketch a graph of $y = g(x)$. For the sketch, you have to approximate the areas under the graphs of f. You can use several rectangles or triangles. Below is the graph of g.



3. Let R be the region in the first quadrant bounded below by the x axis, bounded on the left by the parabola $y = -x^2 + 4x$ and bounded on the right by the line $y = -2x + 5$.

   (a) Sketch the region R.

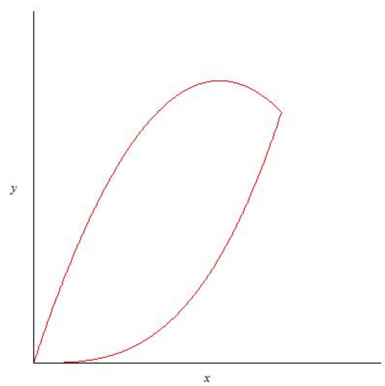
   (b) Estimate the area of the region with $n = 5$ and left points.
   Here $\Delta x = 1/2$. If $f(x) = -x^2 + 4x$ and $g(x) = -2x + 5$ the area estimate is

$$[f(0) + f(1/2) + f(1) + g(3/2) + g(2)] \times \frac{1}{2} = \frac{31}{8}.$$

   (c) Find the exact area of the region. What was you percentage error in your estimation above? (5 points)

$$\int_0^1 -x^2 + 4x \; dx + \int_1^{5/2} -2x + 5 \; dx = \frac{47}{12}$$

4. Let R be the region bounded by the curves $y = x^3$ and $y = 3x - 2x^2$ in the first quadrant as shown below.



   (a) Set up and integral to find the volume of the region obtained by rotating the region R about the y axis. Do not evaluate the integral.
   Using cylndrical shells the volume is

$$\int_0^1 2\pi x \left[(3x - 2x^2) - x^3\right] dx$$

   (b) Set up and integral to find the volume of the region obtained by rotating the region R about the line $y = 2$. Do not evaluate the integral.
   Using disks/washers the volume is

$$\int_0^1 \pi \left[\left(2 - x^3\right)^2 - \left(2 - (3x - 2x^2)\right)^2\right] dx$$