Math 125, Sections D and E, Midterm I

January 28, 2016

Name Solutions

TA/Section

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in you note sheet with your exam.
- You can use a Ti-30x IIS calculator. Put away all other electronic devices.
- For your integrals you may use the following formulas. Anything else must be justified by your work.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \qquad \int e^x\,dx = e^x + C \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C \qquad \int \sec^2 x\,dx = \tan x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C \qquad \int \sec x \tan x\,dx = \sec x + C = \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}dx = \tan^{-1} x + C$$

- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me.

Question	points
1	
2	
3	
4	
Total	

1. Evaluate the following integrals.

(a) (3 points) $\int \frac{x+3}{\sqrt{x+2}}dx$

$u = x+2 \longrightarrow x = u-2$

$du = dx$

$\int \frac{x+3}{\sqrt{x+2}} dx = \int \frac{u-2+3}{\sqrt{u}} du = \int u^{1/2} + u^{-1/2} du$

$= \frac{2}{3} u^{3/2} + 2u^{1/2} + C = \frac{2}{3}(x+2)^{3/2} + 2\sqrt{x+2} + C$

(b) (3 points) $\int_0^1 x(5x^2+1)^2 dx$

$u = 5x^2+1$

$du = 10x\,dx$

$\int_0^1 x(5x^2+1)^2 dx = \int_1^6 \frac{1}{10} u^2 du$

$= \frac{u^3}{30}\Big|_1^6 = \frac{215}{30} = \frac{43}{6}$

$\underline{\underline{OR}}$

$\int_0^1 x(5x^2+1)^2 dx = \int_0^1 x(25x^4+10x^2+1) dx$

$= \int_0^1 25x^5 + 10x^3 + x \, dx$

$= 25\frac{x^6}{6} + \frac{10x^4}{4} + \frac{x^2}{2}\Big|_0^1$

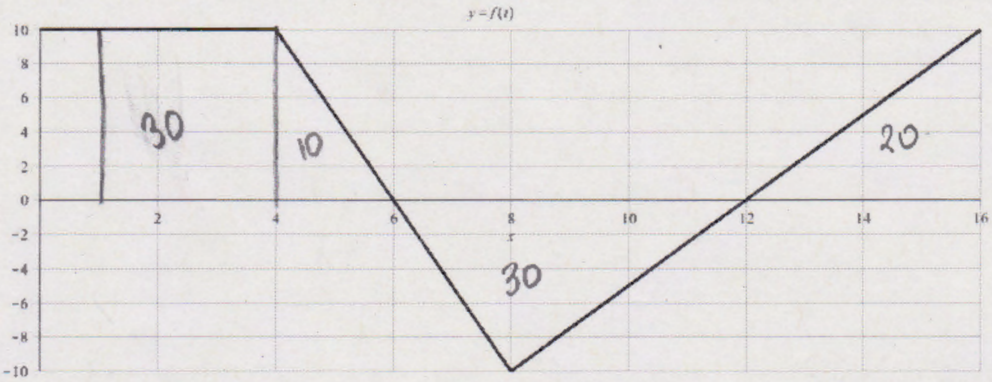
$= \frac{25}{6} + \frac{5}{2} + \frac{1}{2} = \frac{43}{6}$

(c) (3 points) $\int_0^5 |x-3|\,dx = \int_0^3 3-x\,dx + \int_3^5 x-3\,dx$

$= 3x - \frac{x^2}{2}\Big|_0^3 + \frac{x^2}{2} - 3x\Big|_3^5$

$= \left(9-\frac{9}{2}\right) - 0 + \left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right) = -6 + \frac{25}{2} = \frac{13}{2}$

2. Define $g(x) = \int_2^{x^2} f(t)dt$ where the graph of $f(t)$ is given below.



(a) (2 points) Compute g(2).

$$g(2) = \int_2^4 f(t)dt = (4-2)(10) = 20$$

(b) (2 points) Express $g(4) - g(1)$ as a definite integral and compute its value.

$$g(4)-g(1) = \int_2^{16} f(t)dt - \int_2^1 f(t)dt = \int_2^{16} f(t)dt + \int_1^2 f(t)dt = \int_1^{16} f(t)dt$$

$$= 30+10-30+20 = 30$$

(c) (4 points) Compute $g'(3)$.

$$g'(x) = f(x^2)2x$$

$$g'(3) = f(9)\cdot 6$$

equation of $f(x)$, $x \geq 8$:

$$y - 0 = \frac{20}{8}(x-12) \to y = \frac{5}{2}(x-12)$$

$$f(9) = \frac{5}{2}(9-12) = \frac{-15}{2}$$

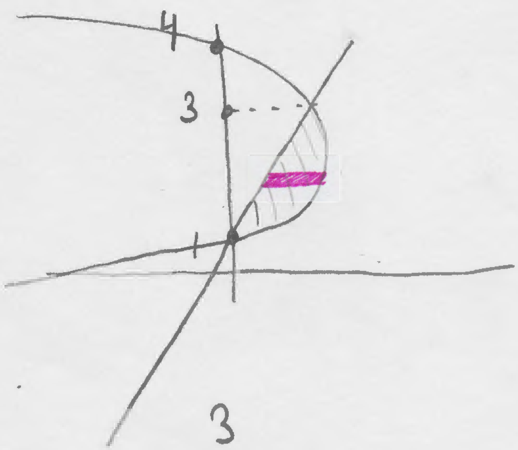
so $g'(3) = -\frac{15}{2} \cdot 6 = -45$

(d) (2 points) Compute $g''(1)$.

$$g''(x) = f'(x^2)2x2x + f(x^2)2$$

$$g''(1) = f'(1)\cdot 4 + f(1)\cdot 2 = 0 + 10\cdot 2 = 20$$

3. (10 points) Find the area of the region to the left of the parabola $x = -(y-1)(y-4)$ and below the line $x = y - 1$. Include a picture of the region.



$-(y-1)(y-4) = y-1$

$y=1$ OR $-(y-4) = 1$

$y = 3$

$$\text{Area} = \int_1^3 -(y-1)(y-4) - (y-1)\,dy$$

$$= \int_1^3 -(y^2-5y+4) - y + 1\,dy$$

$$= \int_1^3 -y^2 + 4y - 3\,dy$$

$$= -\frac{y^3}{3} + 2y^2 - 3y \Big|_1^3$$

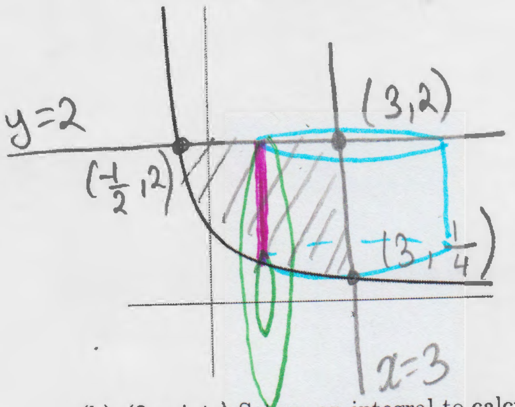
$$= (-9 + 18 - 9) - \left(-\frac{1}{3} + 2 - 3\right)$$

$$= \frac{4}{3}$$

$y = \frac{1}{1+x}$

4. The region **R** is bounded by the hyperbola [redacted], the vertical line $x = 3$ and the horizontal line $y = 2$.

(a) (3 points) Show the region on the graph below and label all intersection points. The hyperbola is given to get you started.



$2 = \frac{1}{1+x}$

$1+x = \frac{1}{2}$

$x = -\frac{1}{2}$

(b) (3 points) Set up an integral to calculate the volume of the solid formed by rotating **R** about the x-axis. Do NOT integrate.

$\Delta V \approx \pi (R^2 - r^2) \Delta x$

$$V = \int_{-1/2}^{3} \pi \left[(2)^2 - \left(\frac{1}{1+x} \right)^2 \right] dx$$

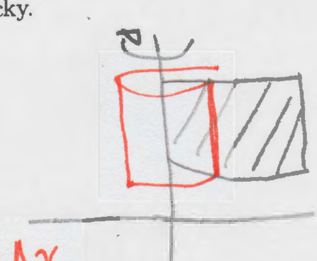
(c) (3 points) Set up an integral to calculate the volume of the solid formed by rotating this region about the vertical line $x = 3$. Do NOT integrate.

$\Delta V \approx 2\pi \text{ (radius) (height) } \Delta x$

$$V = \int_{-1/2}^{3} 2\pi (3-x)\left(2 - \frac{1}{1+x}\right) dx$$

(d) (2 points) Set up an integral to calculate the volume of the solid formed by rotating this region about the y-axis. Do NOT integrate. This is a bit tricky.

This is the same as:
(note the new region)



$\Delta V \approx 2\pi \text{ (radius)(height) } \Delta x$

$$V = \int_{0}^{3} 2\pi \cdot x \cdot \left(2 - \frac{1}{x+1}\right) dx$$