Math 126, Section C, Autumn 2012, Solutions to Midterm I

1. (a) From **v**+**u**= $\vec{AC}$ $=< 2, 6, 2 >$ and **v**-**u**= $\vec{BD}$ $=< 4, 0, -2 >$ we have 2**v**=$< 6, 6, 0 >$ and **v**=$< 3, 3, 0 >$ and **u**=$< 2, 6, 2 > - < 3.3.0 >=< -1, 3, 2 >$.

   (b) From $< 2, 6, 2 >= \vec{AC}$ we get $C = (2, 8, 1)$. From $< -1, 3, 2 >= \vec{AB}$ we get $B = (-1, 5, 1)$.

   (c)
$$\vec{AE} = \mathbf{proj_v u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v}$$
$$= \frac{< -1, 3, 2 > \cdot < 3, 3, 3 >}{< 3, 3, 0 > \cdot < 3, 3, 0 >} < 3, 3, 0 >=< 1, 1, 0 >$$
   so $E = (1, 3, -1)$.

2. (a) The direction vector for the line is $\mathbf{n_1} \times \mathbf{2_2}$=$< 2, -1, 1 > \times < 3, 2, 1 >=< -1, 5, 7 >$.

   To compute a point on both planes we can take $x = 0$ and solve $0 - y + z = 5$ and $0 + 2y - z = 3$ to get $y = 8$ and $z = 13$ so the point is $(0, 8, 13)$.

   Therefore the line equations are
$$x = 0 - t, y = 8 + 5t, z = 13 + 7t.$$

   (b) The normal for the plane is **n**=**v**$\times\vec{PQ}$ where $Q = (0, 8, 13)$. So the normal vector is $< -1, 5, 7 > \times < 0, 1, 11 >=< 48, 11, -1 >$. Therefore, the plane equation is
$$48x + 11y - z = 75.$$

   (c) The same as the line in part (a)

3. (a) $\mathbf{r_1}(t) = \langle t + 3, 2t - 1, -t + 4\rangle$ F $\quad$ $\mathbf{r_2}(t) = \langle 2t + 3, 2t - 1, t + 4\rangle$ E $\quad$ $\mathbf{r_3}(t) = \left\langle t\cos(t), t, \frac{t\sin(t)}{2}\right\rangle$ D

   $\mathbf{r_4}(t) = \langle t, \sin(t), 0\rangle$ B $\quad$ $\mathbf{r_5}(t) = \left\langle t + 1, 2t^2 - 5t + 1, t^3\right\rangle$ E $\quad$ $\mathbf{r_6}(t) = \left\langle \cos(t), 10t, \frac{\sin(t)}{2}\right\rangle$ F

   (b) Plugging in $t = 2$, the point is $(2 + 1, 8 - 10 + 1, 8) = (3, -1, 8)$. The drection vector is $\mathbf{r}'(2)$. We compute $\mathbf{r}'(t) =< 1, 4t - 5, 3t^2 >$ so $\mathbf{r}'(2) =< 1, 3, 12 >$. Therefore the line equation is $\mathbf{r}(t) =< 3 + t, -1 + 3t, 8 + 12t >$.

4. (a) Completing the square and rearranging terms we get
$$\frac{x^2}{4} + z^2 = (y - 1)^2$$
   which is a double cone with the y axis running through it. The two cones meet at the point $(0, 1, 0)$.

   (b) We solve
$$(8t)^2 - 4(5t + 1 - 1)^2 + 4(3 - t)^2 = 0$$
   simplifying we get
$$-8t^2 - 6t + 9 = 0$$
   so $t = -3/2$ or $t = 3/4$, giving us the points $(-12, -13/2, 9/2)$ and $(6, 19/4, 9/4)$.

   (c) Here any three functions $f(t)$, $g(t)$, $h(t)$ with
$$\frac{f(t)^2}{4} + h(t)^2 = (g(t) - 1)^2$$
   would give a curve on the cone. For example $\mathbf{r}(t) =< 2t, t+1, 0 >$ or $\mathbf{r}(t) =< 2t\cos t, t+1, t\sin t >$.