Math 126, Section D, Winter 2010, Midterm I

January 28, 2010

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Instructions.

- There are 4 questions. The exam is out of 40 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.

- You may use a calculator which does not graph and which is not programmable. Even if you have a calculator, give me exact answers. ($\frac{2\ln 3}{\pi}$ is exact, 0.7 is an approximation for the same number.)

- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me. Please $\boxed{\text{BOX}}$ your final answer.

Question	points
1	
2	
3	
4	
Total	

1. Given the two linear vector functions $\mathbf{r}_1(t) =< 2 - t, 3 + 5t, 6t >$ and $\mathbf{r}_2(s) =< 3 + s, 1 + 4s, -2 + 3s >$, answer the following questions about the lines they trace in space.

   (a) Show that the two lines are skew. That is they do not intersect and they are not parallel. (3 points)

   (b) Find the distance between them. (4 points)

   (c) The two skew lines lie on parallel planes. Find the equations of these two planes.(3 points) .

2. Write $< 2, 3, 5 >$ as a sum of two vectors $\mathbf{v}$ and $\mathbf{w}$; $\mathbf{v}$ parallel to $< 1, 2, -1 >$ and $\mathbf{w}$ normal to $< 1, 2, -1 >$. (6 points)

3. Given the vector function given by

$$\mathbf{r}(t) =< t^2 + 5, 3t^3 + 2 >$$

answer the following.

(a) Determine the concavity at the point $(6, -1)$. (6 points)

(b) Find the length of the curve for $0 \le t \le 2$. (2 points)

4. Answer the following.

   (a) Match the following curves in space with their graphs by identifying the surface (give the equation) they are on. (8 points)

   I. $x = t, y = \sin(3t), z = \cos(3t)$ Surface: Graph:

   II. $x = t\sin(5t), y = t\cos(5t), z = t$ Surface: Graph:

   III. $x = \sin(2t), y = \cos(2t), z = \cos(7t)$ Surface: Graph:

   IV. $x = \cos(t)\sin(3t), y = \sin(t)\sin(3t), z = \cos(3t)$ Surface: Graph:
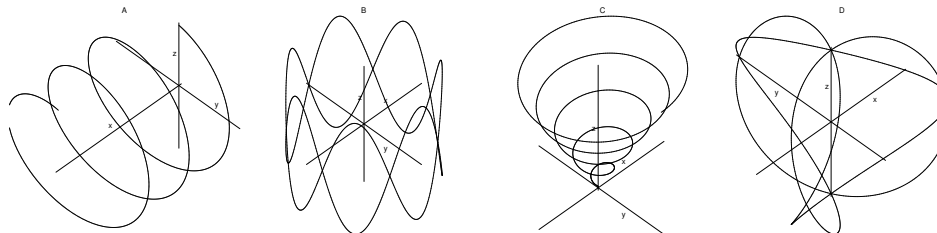
   (b) Find parametric equations for the tangent line to $\mathbf{r}(t) =< \sin(2t), t^2 + 1, \ln(t+1) >$ at the point $(0, 1, 0)$. (6 points)