Math 126 Sections A and B Midterm I

January 30, 2020

Name____

Student Number____________________________________

Instructions.

- These exams will be scanned. **Please write your name and student number clearly for easy recognition.** Answer each question in the space provided. If you absolutely have to use the back page, make a note for us so we can check your work there.
- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and 1/3 are exact, 0.699 and 0.333 are approximations for the those numbers.)
- **Show your work**. If we cannot read or follow your work, we cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you have read all the directions, put a smiley next to your student number for a bonus point.

Question	points
1	
2	
3	
4	
Total	

1. (8 points) The line through the points $P(2,4,-2)$ and $Q(2,-1,3)$ is tangent to the sphere with center $C(1,0,-2)$ and radius 3. Find the equation of the sphere, vector equation for the line, and the point of tangency.

2. (10 points) The two parts of this question are not related.

(a) Determine if the following are True or False. You do not have to justify your answers.

- **T** **F** Two lines parallel to the same plane are parallel.

- **T** **F** For any two vectors $\mathbf{v}$ and $\mathbf{w}$, we always have $\mathbf{v} \cdot \mathbf{w} \times \mathbf{v} = 0$.

- **T** **F** The point $P(1, -3, 4)$ is on the plane $2x - y + z = 9$.

- **T** **F** The lines $\mathbf{r_1}(t) = \langle 2 - t, 1 + 3t, 5 + 2t \rangle$ and $\mathbf{r_2}(t) = \langle 4 + 2t, -5 - t, 1 + 3t \rangle$ are parallel.

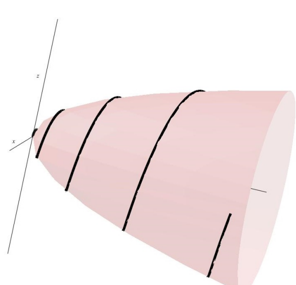
- **T** **F** The points $P(1, 0, 3)$, $Q(3, 2, 1)$ and $R(0, 3, -2)$ are on the same line.

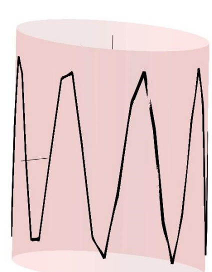
- **T** **F** The line $\mathbf{r}(t) = \langle 1 + 3t, -3t, 1 - 2t \rangle$ is on the plane $4x + 2y + 3z = 7$.

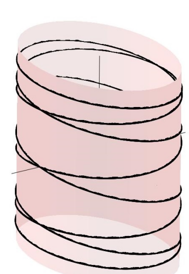
(b) Under each picture, write down the name of the curve, the equation of the surface graphed with the curve and the name of the surface. For example, $\mathbf{r_5}$, $x = 7$, plane. You have to get all three right to get the point for that part. The z-axis points up in all pictures.

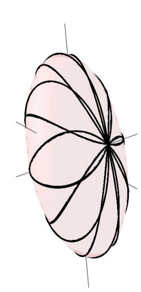
$\mathbf{r_1}(t) = \langle 2\cos(t), 3\sin(t), 5\sin(8t) \rangle$ $\qquad$ $\mathbf{r_2}(t) = \langle 2\cos(8t), 3\sin(8t), 5\sin(t) \rangle$

$\mathbf{r_3}(t) = \left\langle 2t\cos(t), 3t^2 + 2, 2t\sin(t) \right\rangle$ $\qquad$ $\mathbf{r_4}(t) = \langle 2\sin(2t)\cos(9t), \sin(9t), 3\cos(2t)\cos(9t) \rangle$









3. (10 points) The following questions are about the triangle with vertices $A(1,0,3)$, $B(2,5,-1)$ and $C(-3,2,6)$.

   (a) Find the angle at vertex A.

   (b) Find the equation of the plane containing the triangle. Give your answer in standard form.

   (c) Drop a perpendicular from vertex A to side BC. Call the point where it intersects BC point E. Compute the vector $\vec{BE}$.

4. (12 points) Given the vector function $\mathbf{r}(t) = \langle t^2 \sin t, 2t, t^2 \cos t \rangle$ answer the following questions about the curve it traces in space.

   (a) Compute the length of the curve from $t = -\pi$ to $t = \pi$.

   (b) Find the vector equation of the tangent line to the curve at the point where $t = \pi$.

   (c) Find the curvature of the point at the point where $t = 0$.