## Math 126 Sections A and B Midterm II

March 3, 2020

Name

## Student Number

## Instructions.

- These exams will be scanned. Please write your name and student number clearly for easy recognition. Answer each question in the space provided. If you absolutely have to use the back page, make a note for us so we can check your work there.
- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in your notes with your exam paper.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ( $\frac{2 \ln 3}{\pi}$ and $1 / 3$ are exact, 0.699 and 0.333 are approximations for the those numbers.)
- Show your work. If we cannot read or follow your work, we cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you have read all the directions, put a smiley next to your student number for a bonus point.

| Question | points |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| Total |  |

1. ( 7 points) Answer the questions based on the contour graph below.

(a) Give the approximate values of the first partials and the signs $(+/-)$ of the second partials below.

$$
\begin{array}{cl}
f_{x}(Q) \approx & f_{y}(P) \approx \\
f_{x x}(Q) & f_{y y}(P)
\end{array}
$$

(b) List the critical points for this function with their approximate coordinates in the form $(x, y)$ according to their types listed:
Local Miminum
Local Maximum
Saddle
2. The two parts below are not related.
(a) (6 points) Compute the values of $z_{x}, z_{y}$ and $z_{x x}$ for the implicit function $2 x^{2} y+3 y z^{2}+4 x y^{2}=9$ at the point $(1,1,1)$.
(b) (5 points) Use linear approximation to estimate the value of $f(0.05,0.97)$ for the function

$$
f(x, y)=\ln (1+x y)+y^{2}+3 x
$$

3. (10 points) Let $D$ be the region in the $x y$-plane between $y=4 x$ and $y=x^{2}$ in the first quadrant that is below the line $y=4$. The function is $f(x, y)=x y+y^{2}$.
(a) Sketch the region $D$ labeling all boundaries and corners.
(b) Set up the integral $\iint_{D} f(x, y) d A$ in $d x d y$ order. You may have to split the region.
(c) Set up the integral $\iint_{D} f(x, y) d A$ in $d y d x$ order. You may have to split the region.
(d) Evaluate $\iint_{D} f(x, y) d A$ using one of your answers above.
4. (12 points) Find the ratio of the volume of the ellipsoid $x^{2}+y^{2}+4 z^{2}=1$ to the volume of the largest rectangular box that can be inscribed inside it by first computing both volumes. (You do not need to do second derivative verification for the optimization part.)

## Volume of Ellipsoid:

## Volume of Box:

