Integration Practice

Here are some hints. There may be more than one way of solving an integral, especially in the last question. My hints are for the first methods that came to my mind. If you used a different method, email me and I will put it as an alternative.

1. Which method or methods would you use for the following integrals? If you are not sure of your method, carry it out to evaluate the integral.

   (a) $\int \frac{1}{x^2+1} dx$

   $$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

   (b) $\int \frac{1}{x^2-1} dx$
   Factor denominator and do partial fractions

   (c) $\int \frac{x}{x^2+1} dx$
   u- substitution: $u = x^2 + 1$

   (d) $\int \frac{x}{x^2-1} dx$
   u- substitution: $u = x^2 - 1$

   (e) $\int \frac{x^2}{x^2+1} dx$
   First do long division.

   (f) $\int \frac{x^2}{x^2-1} dx$
   First do long division.

2. Which method or methods would you use for the following integrals? If you are not sure of your method, carry it out to evaluate the integral.

   (a) $\int \frac{1}{\sqrt{1-x^2}}dx$

   $$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C$$

   (b) $\int \frac{x}{\sqrt{1-x^2}}dx$

   u- substitution: $u = 1 - x^2$

   (c) $\int \frac{x^2}{\sqrt{1-x^2}}dx$

   trig substitution $x = \sin\theta$

3. Try to evaluate the first integral. If you can do it, great! If not, skip onto the next one for inspiration.

   (a) $\int \sin x \cos x e^{\sin^2 x} dx$

   Do u- substitution: $u = \sin x$ and then see the next question

   (b) $\int x e^{x^2} dx$

   Do u- substitution: $u = x^2$ and then see the next question

   (c) $\int e^x dx$

$$\int e^x dx = e^x + C$$

   Bonus: $\int_{-1}^{1} x e^{x^4} dx$

   The integrand is an odd function.

4. Try to evaluate the first integral. If you can do it, great! If not, skip onto the next one for inspiration.

   (a) $\int \tan^3 x \sec^2 x e^{\tan^2 x} dx$
   Do u- substitution: $u = \tan x$ and then see the next part

   (b) $\int x^3 e^{x^2} dx$
   Do u- substitution: $u = x^2$ and then see the next part

   (c) $\int x e^x dx$
   Integration by parts

5. Try to evaluate the first integral. If you can do it, great! If not, skip onto the next one for inspiration.

(a) $\int \frac{2x+3}{\sqrt{5+4x-x^2}}dx$

Complete the square in the denominator and see the next part.

(b) $\int \frac{5x+13}{\sqrt{4-x^2}}dx$

$$\int \frac{5x+13}{\sqrt{4-x^2}}dx = \int 5\frac{x}{\sqrt{4-x^2}}dx + 13\int \frac{1}{\sqrt{4-x^2}}dx$$

For the first integral you do a u- substitution. For the second one a trig-substitution.

(c) $\int \frac{3x+7}{\sqrt{1-x^2}}dx$

$$\int \frac{3x+7}{\sqrt{1-x^2}}dx = 3\int \frac{x}{\sqrt{1-x^2}}dx + 7\int \frac{1}{\sqrt{1-x^2}}dx$$

See Question 2

6. Try to evaluate the first integral. If you can do it, great! If not, skip onto the next one for inspiration.

(a) $\int \frac{2x+7}{x^2+6x+13}dx$

Complete the square in the denominator and see the next part.

(b) $\int \frac{-x+1}{x^2+9}dx$

$$\int \frac{-x+1}{x^2+9}dx = -\int \frac{x}{x^2+9}dx + \int \frac{1}{x^2+9}dx$$

The first one is u-substitution. The second one is trig substitution.

(c) $\int \frac{7x-2}{x^2+1}dx$

$$\int \frac{7x-2}{x^2+1}dx = 7\int \frac{x}{x^2+1}dx - 2\int \frac{1}{x^2+1}dx$$

See question 1

7. Try to evaluate the first integral. If you can do it, great! If not, skip onto the next one for inspiration.

(a) $\int \frac{x^4+2}{x^3+2x^2+x}dx$
Do long division and see the next part.

(b) $\int \frac{x+2}{x^3+2x^2+x}dx$
Factor the denominator and see the next part.

(c) $\int \frac{7x-2}{x(x+1)(x-3)}dx$
Partial fractions.

(d) $\int \frac{23}{x+1}dx$

8. Evaluate the following integrals.

(a) $\int x \tan^{-1} x dx$
Integration by parts.

(b) $\int_{-1}^{3} |x^2 - 4| dx$
Use the definition of absolute value.

(c) $\int \cos^3(5x) \sin^2(5x) dx$
Do a u-substitution and see page 484 in your textbook.

(d) $\int \ln(x^2 - 5x) dx$
Use the law of logarithms to simplify the integrand. Then use integration by parts.

(e) $\int \frac{1}{\sin^2 x + \cos 2x} dx$
Use a trig identity to simplify the integrand.

(f) $\int \frac{1-\tan\theta}{1+\tan\theta} d\theta$
Use

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and then simplyfy the quotient to use a u-substitution.

(g) $\int_{\pi/4}^{\pi/3} \frac{\sqrt{\tan\theta}}{\sin(2\theta)} d\theta$
Use a trig identity for the denominator and multiply the numerator and denominator by $\cos\theta$. Simplify (tangents and secants) and then do a u-substitution.

(h) $\int_0^1 \frac{xe^{2x}}{(1+2x)^2} dx$
First do partial fractions for $\frac{x}{(1+2x)^2}$. Then try integration by parts for the term with the squared denominator.