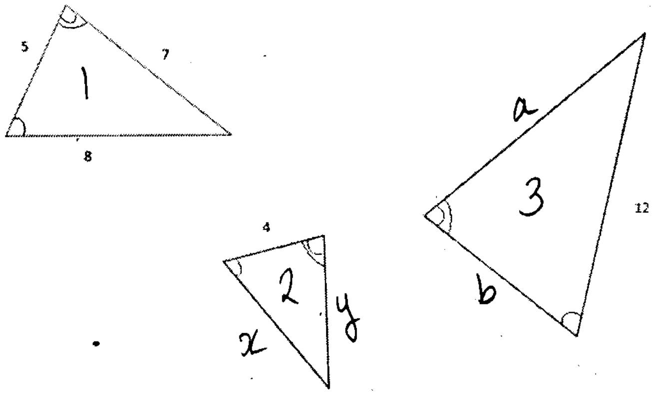
Triangle Geometry – *Solutions*

Three things you have to remember about triangles:

1. **The area of a triangle** is $\frac{1}{2} \times \text{base} \times \text{height}$. The height should be drawn to that base. So potentially there are 3 ways you can compute the area of a triangle, using the 3 pairs of heights and bases.
2. **Pythagorean Theorem** for right triangles.
3. Ratios of corresponding sides for **similar triangles**.

Below are several problems. Most use similar triangles and the ideas or the pictures will appear in story problems throughout this course and Math 124/5.

1. All three triangles below are similar with two of the matching angles marked. Find all missing sides.

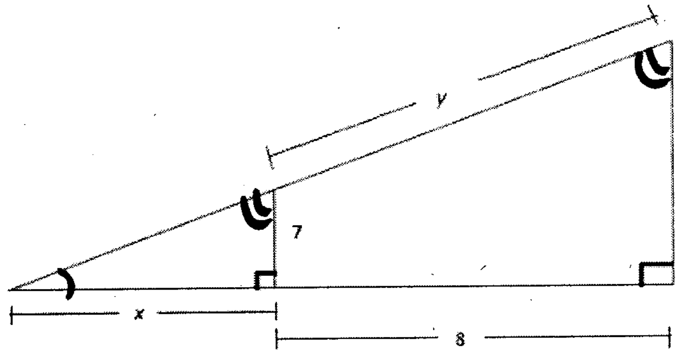


Triangle 1 / Triangle 2: $\frac{5}{4} = \frac{7}{y} = \frac{8}{x}$

so $y = \frac{28}{5}$ and $x = \frac{32}{5}$

Triangle 1 / Triangle 3: $\frac{5}{b} = \frac{7}{a} = \frac{8}{12}$ so $a = \frac{7\times12}{8} = \frac{21}{2}$ and $b = \frac{5\times12}{8} = \frac{15}{2}$

2. The two right angles are marked in the picture. Use similar triangles to find x. Then use the Pythogorean Theorem to find y.



	Horizontal Legs		Vertical Legs
Small	x	=	7
Big	$x+8$		11

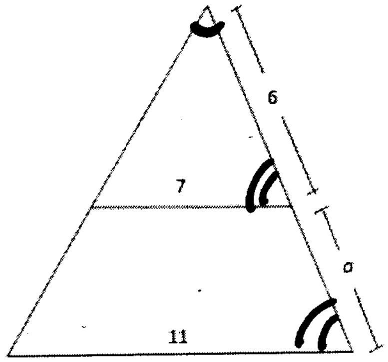
$\frac{x}{x+8} = \frac{7}{11}$ so $11x = 7x + 56$ or $x = \frac{56}{4} = 14$

The hypotenuse of the small triangle is $\sqrt{7^2+14^2} = 7\sqrt{5}$

The hypotenuse of the right triangle is $\sqrt{11^2+22^2} = 11\sqrt{5}$

So, $y = 11\sqrt{5} - 7\sqrt{5} = 4\sqrt{5}$.

3. The two edges with lengths 7 and 11 are parallel. Use similar triangles to find a.



small Δ / Big Δ

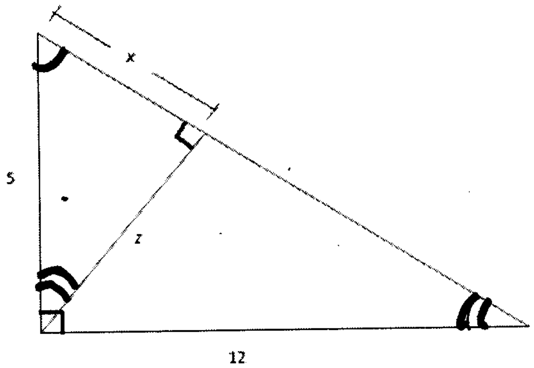
right side / base

$$\frac{6}{6+a} = \frac{7}{11}$$

so $42+7a=66$

$a = \frac{24}{7}$.

4. The two right angles are marked in the picture.



(a) Find the hypotenuse of the big triangle: $\sqrt{12^2+5^2} = 13$

(b) Find x using similar triangles.

small ◺ / big ◺

legs / hypotenuses

$$\frac{x}{5} = \frac{5}{13}$$

so $x = \frac{25}{13}$

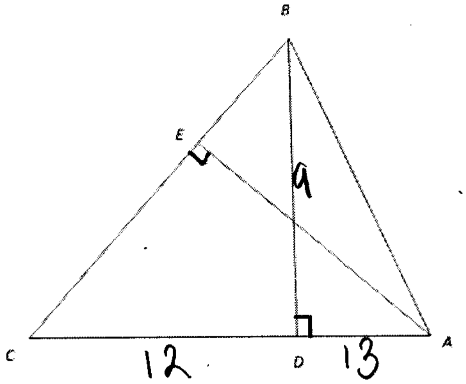
(c) Find z by computing the area of the big triangle in two ways.

$$\frac{5\cdot 12}{2} = \frac{z\cdot 13}{2} \quad \text{so } z = \frac{60}{13}$$

(d) Check that your values for x and z and 5 satisfy the Pythagorean Theorem.

$$\sqrt{x^2+z^2} = \sqrt{\frac{25^2}{13^2}+\frac{60^2}{13^2}} = \frac{1}{13}\sqrt{5^2(5^2+12^2)} = 5$$

5. The right angles are marked. We know: $CD = 12$, $DA = 13$, $BD = 9$. Find EA using the area of the triangle.

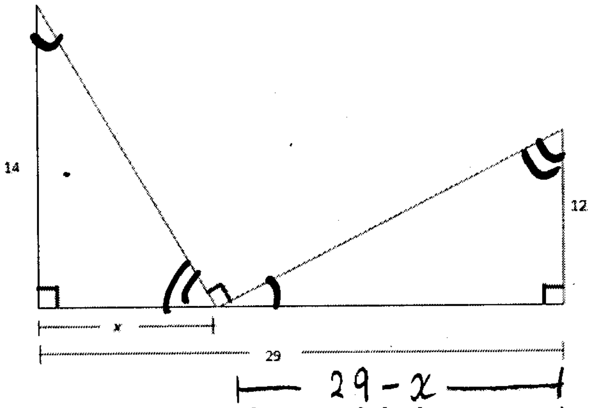


$BC = \sqrt{12^2+9^2} = 3\sqrt{4^2+3^2} = 15$

$\frac{BD \times AC}{2} = \text{Area} = \frac{BC \times EA}{2}$

$\frac{9 \cdot 25}{2} = \frac{15 \cdot EA}{2}$ so $EA = \frac{9 \cdot 25}{15} = 15$

6. The right angles are marked. Find x and the other three missing lengths in the picture.



leg   other leg

Left △ $\frac{14}{29-x} = \frac{x}{12}$ Right △

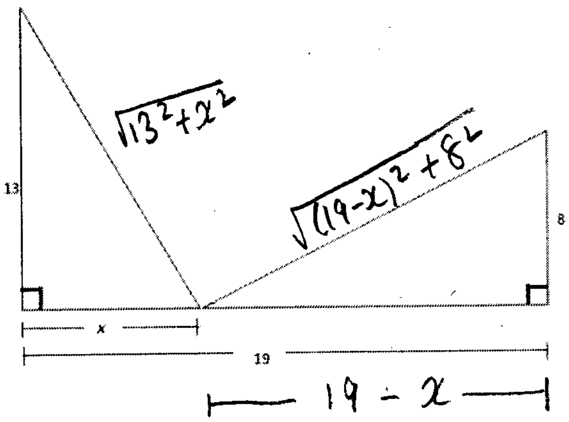
so, $168 = 29x - x^2$

or $x^2 - 29x + 168 = 0$

$x = \frac{29 \pm \sqrt{29^2 - 4(168)}}{2} = \frac{29 \pm 13}{2}$

since $x < 29$, $x = \frac{29-13}{2} = 8$.

7. Find an expression for the sum of the hypotenuses in terms of x. This time the third angle is not necessarily a right angle.



Sum of the hypotenuses $= \sqrt{13^2+x^2} + \sqrt{(19-x)^2+8^2}$