Math 120 Optimization Review

An optimization problem is when you have to maximize or minimize a function you set up from a story. In Math 120, we know how to find minimum or maximum values of quadratic functions using the vertex formula so in the problems below, we try to come up with a quadratic function. Next quarter in Math 124, you will be able to maximize or minimize other functions as well. Below are some example problems with hints to get you started. After solving these, you can try a similar question from the final exam archives.

There are two steps:

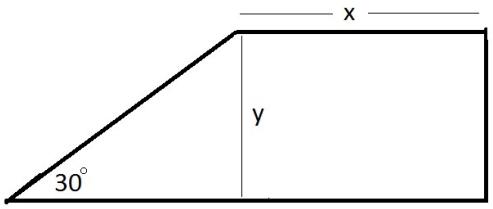
1. Come up with the quadratic function from the story.
2. Find its vertex and answer the question.

It is the first step that takes most effort which may involve geometry.

Problems

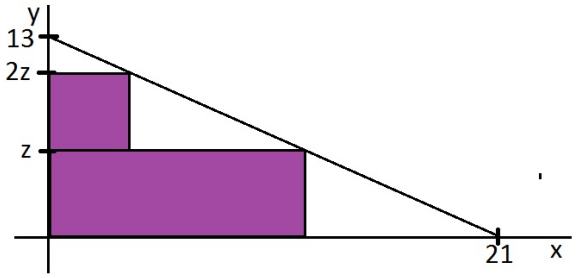
1. Two robots Amy and Bob are moving on the coordinate plane. Amy starts at $(0, 4)$ and goes stright at constant speed towards the point $(10, -1)$ reaching it in 5 seconds. Bob starts at the same time as Amy at $(1, -1)$ and goes stright at constant speed towards the point $(10, 2)$ reaching it in 3 seconds. When are they closest together? What is the shortest distance between them?

2. You have 150 cm of wire to make the following shape:

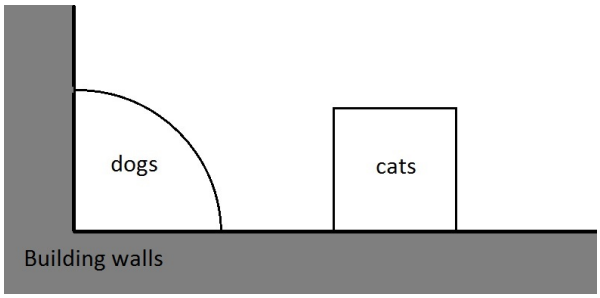


   Find the length of the sides x and y that maximizes the area of the trapezoid.

3. Find the value of z which maximizes the purple area below.



4. You are builing fenced enclosures to the corner of a building as shown. The dogs will be kept in the enclosure that has the shape of a quarter of a circle and they need a 6 foot wire fence which costs 1.2 dollar per foot. The cats will be in the square enclosure which has to be a wooden fence costing 2.8 dollars per foot. Find the maximum total area possible when you have 250 dollars to spend.



5. In the market economy, the lower the price, more people are likely to buy the item. Assume that the number of handbags sold $H(p)$ is a linear function of their price p. If a particular style of handbag is \$120, 23 will buy it on Black Friday at Nordstrom. If it is priced at \$160, 15 people will buy it. In order to maximize the revenue on that day, what price should Nordstrom set on that particular bag?

Hints, Answers and Suggested Problems

1. First, you write down the parametric equations of their motion.

$$x_A = 2t, y_A = 4 - t, x_B = 1 + 3t, y_B = -1 + t$$

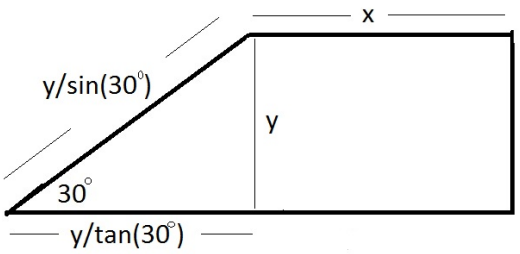
Then, you write down the distance formula.

$$\text{distance} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{((2t) - (1 + 3t))^2 + ((4 - t) - (-1 + t))^2}$$

When you simplify, you will see that the part inside the square root is quadratic. You find the t which minimizes it and answer the question.

$t = 1.8$, minimum distance $= \sqrt{9.8}$. For a similar problem, try Question 7 in Winter 2015.

2. First, set the circumference equal to 150 using the hint below.



Then, write down the area.

$$A = \frac{y^2}{2\tan(30°)} + xy$$

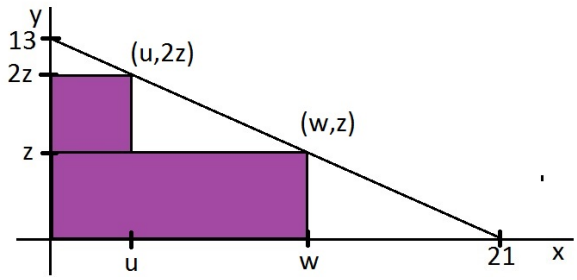
Now use the circumference equation to get rid of x or y. Then, you'll have your quadratic function to maximize.

$y = 25, x = \frac{75-25\sqrt{3}}{2} \approx 15.85$. For a similar question see Question 6 in Winter 2014.

3. The area is

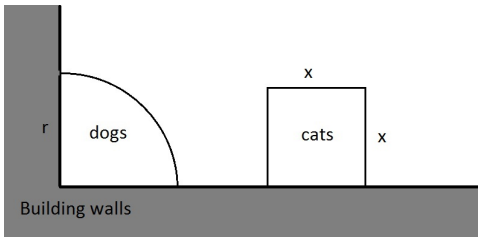
$$A = uz + wz$$

Now, use the fact that the points shown are on the line through $(0, 13)$ and $(21, 0)$ to compute u and w in terms of z. Write your area as a function of z only.



$z = 13/3$. For a similar question see Question 6 in Autumn 2014.

4. Use the variables to compute the total cost of the fences and set it equal to 250. Solve from x or r from this. Now write down the area in terms of x and r and eliminate one of your variables using your cost equation.



$r = \frac{150}{17.64+0.36\pi} \approx 7.99$, Area ≈ 832.42. For similar question see Question 6 in Winter 2013 or Question 3 on Autumn 2012.

5. The total revenue is the number of handbags sold times the unit price so you want to maximize $R = pH(p)$.

$p = \$117.5$. For a similar question, see Question 2 in Autumn 2013.