## Math 120, Sections A and B, Fall 2015, Midterm I

October 22, 2015

Name


TA/Section

## Instructions.

- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in you note sheet with your exam.
- You can use a TI 30X-IIS calculator. Put away all other electronic devices.
- Show your work. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me.
- You are expected to give an EXACT answer to all of the questions. The numbers $\frac{1}{3}, 1+\sqrt{2}, \pi$ are exact. The numbers $0.3333333,2.414$, and 3.1416 are decimal approximations for them. A decimal does not have to be an approximation. For example, $\frac{1}{4}=0.25$ or $\frac{12}{10}=1.2$. In that case, using one or the other is your preference.

| Question | points |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| Total |  |

1. (12 points) A sheet of metal is in the shape shown below. Let $x$ be the width of the shaded area. The picture is not to scale.


Write down a multi-part function $A(x)$ giving the shaded area as a function of $x$. Note that there are two possible area shapes depending on the value of $x$, shown by the two pictures above.
when $0 \leq x \leq 9, \quad A(x)=7 x$
When $9 \leqslant x \leqslant 14$
similar triangles: $\frac{x-9}{5}=\frac{y}{14}$ so $y=\frac{14(x-9)}{5}$

$$
A(x)=7 x+\frac{y(x-4)}{2}=7 x+\frac{14(x-9)^{2}}{10}
$$

So,

$$
A(x)= \begin{cases}7 x, & , 0 \leq x \leq 9 \\ 7 x+\frac{14(x-9)^{2}}{10}, & , 9<x \leq 14\end{cases}
$$

2. Two dogs and their owners are at a dog park. Romeo starts running at 35 feet North of the tree towards its owner Juliet who is standing at 200 feet East and 135 feet North of the tree. Running at constant speed, Romeo reaches Juliet in 10 seconds. At the same time when Romeo starts running, Buddy starts running from a point 95 feet North of Romeo and heads straight for its owner Charlie who is standing 120 feet East of the tree, also ruuuing at constant speed. Buddy crosses Romeo's path exactly secondsafter he has passed the same spot - so they do not run into each other. +wo
(a) (4 points) Impose a coordinate system with the tree as the origin and find parametric equations for Romeo's motion. Below is where they are at $t=0$ and their paths to get you started. The picture is not to scale.


$$
\begin{array}{cl}
x_{R}=a+b t & y_{R}=c+d t \\
\left.0=x_{R} \mid 0\right)=a & 35=y_{R}(0)=c \\
200=x_{R}(10)=0+106 & 135=35+10 d \\
20=b & 10=d \\
x_{R}=20 t & y_{R}=35+10 t
\end{array}
$$

(b) ( 6 points) Find the point where their paths intersect.

$$
\begin{aligned}
& (6 \text { points) Find the point where their paths intersect. } \\
& \text { Rome's path: } \quad m=\frac{135-35}{200-0}=\frac{1}{2}
\end{aligned}
$$

Buddy's path: $m=\frac{0-130}{120-0}=\frac{-13}{12}$ so $y=\frac{-13}{12} x+130$
Point of intersection

$$
\begin{aligned}
& \text { ersection } \\
& \frac{1}{2} x+35=\frac{13}{12} x+130
\end{aligned}
$$

(c) (4 points) Find parametric equations for Buddy's motion.

$$
\begin{aligned}
& \left(\frac{1}{2} x+35=\frac{13}{12} x+130\right. \\
& \left(\frac{6}{12}+\frac{13}{12}\right) x=95 \text { so } x=\frac{12(95)}{19}=60 \quad y=\frac{1}{2} \times 60+35=65 \\
& (60,65)
\end{aligned}
$$

$$
\begin{array}{rl}
x_{B}=a+b t & y_{B}=c+d t \\
0=x_{B}(0)=a & 130=y_{B}(0)=c
\end{array}
$$

Romeo reaches $(60,65)$ when $20 t=60$ or $t=3$

$$
\begin{aligned}
& \text { Romeo reaches }(60,65) \\
& \text { so buddy reaches }(60,65) \text { when } \\
& \qquad 60=x_{B}(5)=0+5 b \\
& \begin{array}{ll}
12=b & 65=y_{B}(5)=130+5 d \\
\text { ie. } & -13=\frac{65}{5}=d \\
\text { so } & x_{B}=12 t \quad y_{B}=130-13 t
\end{array}
\end{aligned}
$$

3. Megan is standing on a downhill street with a $10 \%$ incline, i. e. it goes down vertically 10 feet for every 100 feet horizontal distance. She kicks the ball up in the air so that the ball follows the path of the graph of

$$
y=-0.1 x^{2}+2 x
$$

where both $x$ and $y$ are in
feet
(a) (1 point) Sketch the path of the ball on the picture below.

(b) (5 points) Where does the ball hit the street? Give your answer in the form $(x, y)$.

Sheet equahon: $y=\frac{-10}{100} x=-0.1 x$
Ball hits street when

$$
\begin{aligned}
& -0.1 x=-0.1 x^{2}+2 x \\
& 0.1 x^{2}-2.1 x=0 \\
& (0.1 x-2.1) x=0 \\
& x=\frac{2.1}{0.1}=21 \text { or } x=0 \\
& y=-0.1 x=-2.1 \text { (STARTS } \\
& (21,-2.1)
\end{aligned}
$$

(c) (4 points) What is the maximum height of the ball, measured from the ground? height of ball $h(x)=\left(-0.1 x^{2}+2 x\right)-(-0.1 x)$

$$
h|x| \text { has max. value when } x=\frac{-2.1}{2(-0.1)}=\frac{21}{2}=10.5
$$

The maximum height is

$$
\begin{aligned}
& \text { xumum height is } \\
& \begin{aligned}
h(10.5) & =-0.1(10.5)^{2}+2.1(10.5) \\
& =11.025 \text { feet }
\end{aligned}
\end{aligned}
$$

4. (14 points) I want to go for a swim in the circular lake of radius 1 mile. Initially, I am at 9 miles East and 12 miles North of the center of the lake. First I run towards the center of the lake so I reach it as soon as possible. Then I go in, swim due and get out of the lake. If I can run at 7 miles per hour and swim at 4 miles per hour, how long does it take me to run and swim altogether?
Hint: Impose a coordinate system where the origin is at the center of the lake.


Irun for $15-1=14$ miles so $\frac{14}{7}=2$ hours.
To find the distance I swim, I need the $x$-cordinak of the point $L$ :
live: $y=\frac{12}{4} x=\frac{4}{3} x \quad$ circle: $x^{2}+y^{2}=1$
intersechon:

$$
\begin{aligned}
& x^{2}+\left(\frac{4}{3} x\right)^{2}=1 \\
& \left(1+\frac{16}{9}\right) x^{2}=1 \\
& \frac{25}{9} x^{2}=1 \text { so } x=\sqrt{\frac{a}{25}}=\frac{3}{5}
\end{aligned}
$$

I scrim for $2 \cdot \frac{3}{5}=\frac{6}{5}$ miles for $\frac{6}{5} \cdot \frac{1}{4}=\frac{6}{20}=\frac{3}{10}$ hours So Total hie is $2+\frac{3}{10}=2.3$ hours.

