Math 120, Sections A and B, Fall 2015, Midterm II

Name _____ $Solutions$ _____

TA/Section _____

Instructions.

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Write your name on your note sheet and hand it with your exam.

- You can use a TI 30X-IIS calculator. Put away all other electronic devices.

- **Show your work**. If I cannot read or follow your work, I cannot grade it. If you are reading this, put a star next to your name for one point bonus. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me.

Question	points
1	
2	
3	
4	
Total	

1. (8 points) A rectangle sits in the first quadrant with one corner on the origin and the opposite corner on the line through the points shown in the graph. Let x be the length of the rectangle as shown. Write down the area of the rectangle as a function of x and find the maximum possible area of such a rectangle.
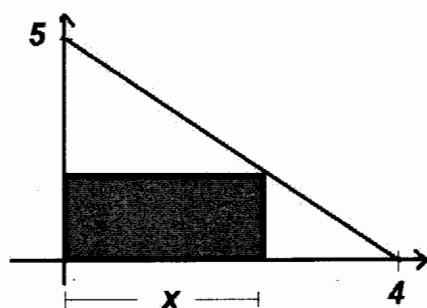
line equation

$$y = -\frac{5}{4}x + 5$$

$$\left] -\frac{5}{4}x + 5 \right.$$

$$A(x) = x\left(-\frac{5}{4}x + 5\right) = -\frac{5}{4}x^2 + 5x$$

vertex at $\quad x = \dfrac{-5}{2\left(-\frac{5}{4}\right)} = 2$

max. area $\quad A(2) = -\frac{5}{4} \cdot 4 + 5 \cdot 2 = 5$

2. (a) (6 points) Match the following equations with their graphs labeled A-D by writing down the letter of the graph next to its equation.
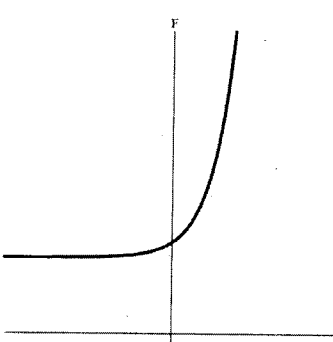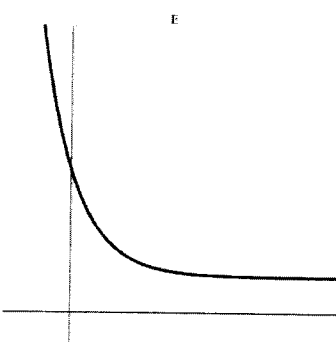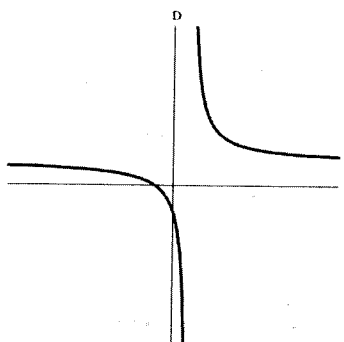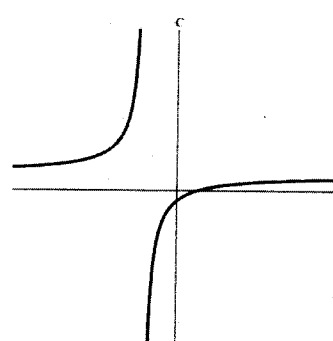
$\underline{E}$ decay $\quad y = 3e^{-2x} + 1$

$\underline{F}$ growth $\quad y = e^{1.2x} + 5$

$\underline{B}$ $\quad y = \dfrac{4 \pm \sqrt{16 - 4(3)(2)}}{6}$ no roots $\quad y = 3x^2 - 4x + 2$

$\underline{C}$ $\quad y = \dfrac{2x - 5}{x + 4}$ VA $x = -4$

$\underline{A}$ $\quad y = 2x^2 - 5x + 2$ $\quad x = \dfrac{5 \pm \sqrt{25 - 16}}{4} = 2, \frac{1}{2}$

$\underline{D}$ $\quad y = \dfrac{3x + 5}{2x - 3}$ VA $x = 3/2$
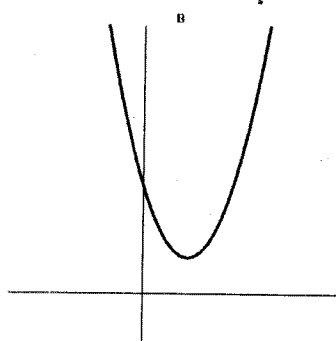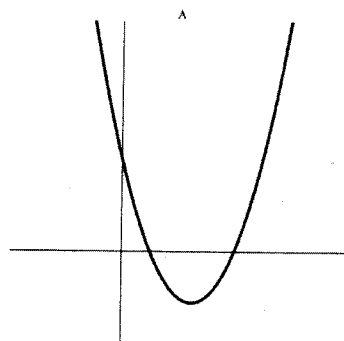
(b) (8 points) Let $f(x) = 3 - e^x$ and $g(x) = \ln x$. Compute $h(x) = g(f(x))$ and find its inverse $h^{-1}(x)$. What is the domain and range of $h(x)$?

$h(x) = \ln(3 - e^x)$ domain: $3 - e^x > 0$

$3 > e^x$

$\boxed{\ln 3 > x}$

$y = \ln(3 - e^x)$

$e^y = 3 - e^x$

$e^x = 3 - e^y$

$x = \ln(3 - e^y)$

$\boxed{h^{-1}(x) = \ln(3 - e^x)}$

domain $\ln 3 > x$
so range of h: $\boxed{y < \ln 3}$

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3. The median household income and median home price in the United States since 1965 can be modeled by exponential functions.

(a) (5 points) In 1965, the median household income was $6,900 and in 1975 it was $11,800. Write down a formula for the median household income $I(t)$ in dollars where t is the number of years since 1965.

$I(0) = 6900$ so $I(t) = 6900 b^t$

in 1975, $11,800 = I(10) = 6900 b^{10}$

so $b = \left(\frac{11800}{6900}\right)^{1/10} \approx$

$$I(t) = 6900 \left(\frac{118}{69}\right)^{t/10} \approx 6900(1.05512)^t$$

(b) (6 points) In 1965, the median home price was 3 times the median household income. By 1995, it was 3.75 times the median household income. Write down a formula for the median home price $P(t)$ in dollars where t is the number of years since 1965.

$P(t) = P_0 b^t$ $P_0 = 3 I_0 = 20700$

$P(t) = 20700 b^t$

in 1995, $3.75 \, I(30) = 20700 \, b^{30}$

$\frac{3.75}{3}\left(\frac{118}{69}\right)^3 = \frac{(3.75)(6900)\left(\frac{118}{69}\right)^{\frac{30}{10}}}{20700} = b^{30} \longrightarrow b = \left[\frac{3.75}{3}\left(\frac{118}{69}\right)^3\right]^{\frac{1}{30}} \approx 1.063$

$$P(t) \doteq 20700(1.063)^t$$

(c) (4 points) In what year was the median home price 4 times the median household income in the United States?

$P(t) = 4 I(t)$

$20700(1.063)^t = 4 \times 6900 \times (1.05512)^t$

$\left(\frac{1.063}{1.05512}\right)^t = \frac{(1.063)^t}{(1.05512)^t} = \frac{4 \times 6900}{20700} = \frac{4}{3}$

$t \ln\left(\frac{1.063}{1.05512}\right) = \ln(4/3) \longrightarrow t = \frac{\ln(4/3)}{\ln\left(\frac{1.063}{1.05512}\right)} \approx 38.7$

In $1965 + 39 = 2004$

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4. (13 points) A bag of popcorn is popping in the microwave. The number of un-popped kernels $f(t)$ is a linear-to-linear rational function of time. After 1 minute, there are 160 un-popped kernels left in the bag. After 3 minutes, there are 120 un-popped kernels left in the bag. If the bag is left in the microwave indefinitely, there will eventually be 60 un-popped kernels left in the bag. How many un-popped kernels were there before the microwave started ?

$$f(t) = \frac{at + b}{t + d}$$

UN-POPPED   **POPPED**

$$f(1) = 160 \qquad f(3) = 120 \qquad \boxed{a = 60}$$

$$\text{So } f(t) = \frac{60t + b}{t + d}$$

$$160 = f(1) = \frac{60 + b}{1 + d} \longrightarrow 60 + b = 160 + 160d \qquad (-1)$$

$$120 = f(3) = \frac{180 + b}{3 + d} \longrightarrow + \frac{180 + b = 360 + 120d}{120 = 200 - 40d}$$

$$40d = 80$$

$$\boxed{d = 2}$$

$$60 + b = 160 + 160(2)$$
$$b = 480 - 60 = 420$$

$$f(t) = \frac{60t + 420}{t + 2}$$

$$f(0) = \frac{420}{2} = 210 \text{ un-popped kernels.}$$

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