Math 135 Final Review

Final Exam Topics

Sequences and series

1. Indeterminate forms, L'Hospital's rule, improper integrals.

2. Sequences: Basic definitions and theorems about convergence, important limits, fixed points of contractions.

3. Series: Basic definitions, important series (geometric, harmonic, p-series), convergence tests (comparison, integral, root, ratio, alternating series), Taylor series and power series, radius of convergence, interval of convergence, important examples (like e^x, $\sin x$, $\cos x$, $\ln x$), differentiation and integration of series, Abel's theorem.

Differential equations.

1. $f_n(t)y^n + ... + f_1(t)y' + f_0(t)y = g(t)$: Linear independence, form of the general solution in both the homogeneous and non homogeneous cases.

2. $a_n y^n + ... + a_1 y' + a_0 y = 0$: Solution using the characteristic equation.

3. $a_n y^n + ... + a_1 y' + a_0 y = f(t)$: Method of undetermined coefficients, Laplace Transforms, variation of parameters.

4. $y'' + p(t)y' + q(t)y = g(t)$: Reduction of order.

   Picard iterations and the formulas from the mass-spring systems will not be on the final. You will have the Laplace Table sheet. Also, I will not ask a series solution question on the final.

3D Space, Vectors and Vector Calculus

1. Vectors, dot and cross products, orthogonal and parallel vectors.

2. Equations of lines and planes in space. Using vectors to calculate distances between points, lines, planes.

3. Definitions and computations of limits, derivatives and integrals of vector functions.

4. Curves: Intersecting curves in space, tangent lines to curves, **T**, **N** and **B** vectors, normal and osculating planes, arc length, curvature.

Review Questions on vectors, 3D space and vector calculus

1. Given the two vector functions $\mathbf{r_1}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ and $\mathbf{r_2}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + t^4\,\mathbf{k}$

   (a) Find the point where the curves traced by the two vector functions intersect.

   (b) Find parametric equations for each of the tangent lines to these curves at their point of intersection.

   (c) Find the angle between the two tangent lines to the curves at that point.

   (d) Find the equation of the plane containing these two tangent lines.

2. Define a curve by

$$\mathbf{r}(t) = \cos 3t\,\mathbf{i} + t\,\mathbf{j} - \sin 3t\,\mathbf{k}.$$

   Find the vectors **T**, **N** and **B** as functions of t.

3. Consider the plane Π: $x + 2y + 3z = 10$

   (a) Show that the line ℓ given by the equation $\mathbf{r}(t) = (4\mathbf{i} + 3\mathbf{j}) + t(3\mathbf{j} - 2\mathbf{k})$ is contained in the plane Π.

(b) Find a parametric equation for the line in the plane Π passing through the point $P(3,2,1)$ on the plane and intersecting the line ℓ orthogonally.

4. Given the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and the plane plane $x + y + 2z = 4$ at a point.

   (a) Find the point where the curve intersects the plane. (This is guess and check.)

   (b) Find the angle between the tangent to the curve and the normal to the plane at that point.

5. Three objects move in space according to the equations

$$\mathbf{r} = \mathbf{r}_1(t) \quad \mathbf{r} = \mathbf{r}_2(t) \qquad \text{and } \mathbf{r} = \mathbf{r}_3(t)\,,$$

   where t denotes time. Let $A(t)$ denote the area of the triangle formed by the three objects. Suppose that

$$\begin{array}{lll} \mathbf{r}_1(0) = \mathbf{i}+\mathbf{j}+\mathbf{k} & \mathbf{r}_2(0) = \mathbf{i}+\mathbf{j}-\mathbf{k} & \mathbf{r}_3(0) = \mathbf{k} \\ \mathbf{r}_1'(0) = \mathbf{i} & \mathbf{r}_2'(0) = \mathbf{j} & \mathbf{r}_3'(0) = \mathbf{k} \end{array}$$

   Compute $A'(0)$.

6. Given the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and the sphere $x^2 + y^2 + z^2 = 3$,

   (a) Find the point where the curve intersects the sphere. Again, use guess and check.

   (b) Find the angle between the tangent to the curve and the normal to the sphere at that point.

   (c) Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 = 3$ at that point.

7. The trajectory of an object is given by the formula $\mathbf{r}(t) = t\,\mathbf{i} + \mathbf{j} + \frac{t^2}{2}\,\mathbf{k}$, where t denotes time.

   (a) Find the speed of the object at time t.

   (b) Find the unit tangent vector $\mathbf{T}$ to the curve traversed by the object as function of t.

   (c) Find both the curvature κ and the unit normal $\mathbf{N}$ to the curve traversed by the object as functions of t.

8. Use vectors to find the distance from the point $P(1,2,3)$ to the line $\mathbf{r}(t) = (2-4t)\mathbf{i} + (-1+3t)\mathbf{j} + 5t\mathbf{k}$