Table of Laplace Transforms

The Laplace Transform of $f(t)$ is defined to be $F(s) = \int_0^\infty e^{-st} f(t)dt$ where it converges. We do not want to evaluate this improper integral each time, so we use the table below. The restrictions on s in the Laplace transforms are omitted. The Theorem numbers refer to lecture notes.

	Function $f(t)$	**Laplace transform** $F(s)$	**How to compute**
1	1	$\frac{1}{s}$	Use the definition of the Laplace transform.
2	e^{at}	$\frac{1}{s-a}$	Use #1 and Theorem 1.
3	$t^n, n = 0, 1, 2, ...$	$\frac{n!}{s^{n+1}}$	Use #1 and Theorem 5.
4	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	Use Theorem 5 and #2 or Theorem 1 and #3.
5	$\cos(at)$	$\frac{s}{s^2+a^2}$	Use the definition.
6	$\sin(at)$	$\frac{a}{s^2+a^2}$	Use #5 and Theorem 4 with $n = 1$.
7	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$	Use Theorem 5 and #5 above.
8	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	Use Theorem 5 and #6 above.
9	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$	Use Theorem 1 and #5 above.
10	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	Use Theorem 1 and #6 above.

Theorem 1 is about shifts of the Laplace transform:

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a), \text{ where } F(s) = \mathcal{L}\{f(t)\}.$$

Theorem 3 is about the Laplace transform of the n^{th} derivative:

$$\mathcal{L}\{y^{(n)}\} = s^n\mathcal{L}\{y\} - y^{(n-1)}(0) - sy^{(n-2)}(0) - s^2y^{(n-3)}(0) - ... - s^{n-2}y'(0) - s^{n-1}y(0).$$

Theorem 4 is about Laplace transforms of shifts:

$$\mathcal{L}\{g(t-c)\alpha(t-c)\} = e^{-cs}\mathcal{L}\{g\}.$$

Theorem 5 is about the derivatives of the Laplace transform $F(s)$:

$$\frac{d^n}{ds^n}F(s) = \mathcal{L}\{(-t)^n f(t)\}, \text{ where } F(s) = \mathcal{L}\{f(t)\}.$$

Theorem 6 is about the product of Laplace transforms:

$$\mathcal{L}^{-1}\{F(s)G(s)\} = \int_0^t g(\beta)f(t-\beta)d\beta$$

where $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$. Finally, we have the Laplace transform of a periodic function $f(t)$ with period P:

$$\mathcal{L}\{f(t)\} = \frac{\int_0^P e^{-st}f(t)dt}{1 - e^{-Ps}}$$