Hints to Midterm 1 Review Questions

1. Look at even and odd n's separately and see if they are increasing or decreasing.
2. Consider the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 1/(x^4 + 16)$.
3. Show that $f(x) = 1 + \displaystyle\int_0^x \frac{\cos(t)}{t^2+4}\,dt$ is a contraction.
4. Show that $f(x) = \pi + \dfrac{1}{2}\sin(x)$ is a contraction.
5. Laws of logarithms.
6. You can do this using the fixed point theorem or you can come up with a formula for a_n and prove it by induction. Then you can take the limit as $n \to \infty$
7. Use comparison.
8. Parts of this was covered in class and homework. Use comparison for the rest.
9. Comparison, integral test, ratio test, root test, alternating series.
10. Section 12.9
11. Recall that $\arcsin(x) = \int_0^x (1-t^2)^{-1/2} dt$. If you wish, you may use the binomial expansion of $(1+x)^{-1/2}$.
12. This example shows that the Taylor series of f about 0 converges everywhere but it only equals f at the origin.
13. Use Taylor's Theorem.
14. Use Taylor's Theorem.
15. Use Taylor's Theorem. This is mostly by trial and error. Try to get the interval as large as possible.
16. Compare the series for $R_n(x)$ with an appropriate geometric series.
17. First check absolute convergence to get the radius of convergence. Then, check the endpoints.
18. This is an indeterminate power.
19. Using series may be fater than L'Hospital's Rule.
20. Some are improper in more than one way.