Math 135, Winter 2015, Homework 1

For practice - do not hand in

1. **Section 11.2**, Problems 7, 17, 30, 35 , 47, 55, 61.

2. **Section 11.3**, Problems 9, 17, 25 (cite the theorems you use in your steps), 47, 63.

3. Let

$$a_1 = \frac{3}{2}, \qquad a_{n+1} = \frac{a_n^2 + 2}{2a_n}$$

Prove that the sequence is bounded below by $\sqrt{2}$ and that it is decreasing. By Theorem 11.3.6 it converges. Find the limit.

4. General formula for a simple linear recursion.

Let

$$a_1 = a, \qquad a_{n+1} = \alpha a_n + \beta$$

Determine the conditions under which a limit exists and find the limit in two different ways:

(a) Guess and check (by induction) an explicit formula for a_n. Take the limit. There may be some conditions on α and β to guarantee a limit.

(b) Use the theorem from Fixed Points handout.

To hand in

1. Problem 52 in Section 11.3.

2. Read the proof of the first part of Theorem 11.3.6 and prove the second part.

3. Suppose that f is a differentiable function on $(0, \infty)$ such that $f'(x) \to 0$ as $x \to \infty$. Show that

$$\lim_{n \to \infty} \left(f(n+1) - f(n)\right) = 0\,.$$

For instance, $\sqrt{n+1} - \sqrt{n} \to 0$ as $n \to \infty$ even though $\sqrt{n} \to \infty$ as $n \to \infty$.

Hint: *Use the Mean Value Theorem.*