Math 135, Winter 2015, Homework 3

For practice - do not hand in

1. **Section 12.4**, Problems 6, 7, 8, 21, 31, 37, 39.
2. **Section 12.5**, Problems 4, 18, 19, 36, 45.
3. Use induction to prove Taylor's Theorem. The case $n = 1$ is Problem 78 in Section 8.2 which was a homework problem last quarter.
4. **Section 12.6**, Problems 9, 14, 29, 45, 50.
5. **Section 12.7**, Problems 9, 14 17, 21, 32.
6. If we are estimating $\ln(x)$ with the 5th Taylor polynomial near 1 and we need the error to be less than 5×10^{-8}, what is the largest set of possible values for x?

To hand in

1. Problem 42 in 12.4.
2. This problem proves a generalization of the Mean Value Theorem (part (c) below). As a corollary, you can show that the Cauchy Formula for the Remainder is a consequence of the Lagrange Formula.
   (a) Problem 45 in Section 4.1.
   (b) Use Cauchy's Mean Value Formula from part (a) to prove that if f and g are continuous on $[a, b]$, then
   $$g(c) \int_a^b f(t)dt = f(c) \int_a^b g(t)dt$$
   for some c in (a, b).
   (c) Prove that if ϕ and h are continuous on $[a, b]$ and $h(t) \neq 0$ for any t in (a, b) then
   $$\int_a^b \phi(t)h(t)dt = \phi(c) \int_a^b h(t)dt$$
   for some c in (a, b).
   (d) Prove that the remainder $\frac{1}{n!} \int_a^x f^{(n+1)}(t)(x-a)^n dt$ in Taylor's Theorem equals $\frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$ for some c between a and x.
   (e) Show that
   $$\frac{1}{10\sqrt{2}} < \int_0^1 \frac{x^9}{\sqrt{1+x}} dx < \frac{1}{10}.$$
3. Problem 66 in Section 12.6. There is a typo: $s_q = \sum_{k=0}^{q} \frac{1}{k!}$.
4. The decreasing condition in the Alternating Series Test (due to Leibniz) is important. Try to find an example where $\lim_{n\to\infty} a_n = 0$, $a_n \geq 0$ but the alternating series $\sum(-1)^k a_k$ diverges. (Problem 42 in 12.5)