Math 135, Winter 2015, Homework 4

For practice - do not hand in

1. **Section 12.8**, Problems 1, 9, 21, 25, 31, 37, 41, 45.
2. **Section 12.9**, Problems 1, 5, 9, 25, 29, 35, 43, 50.
3. Find the 5th degree Taylor Polynomial for the following functions:
   (a) $(x^3+1)e^{2x}$.
   (b) $\sin\left(x+x^2\right)$.
   (c) $\cos\left(e^x-1\right)$.
4. **Section 10.2**, Problems 5, 15, 23, 40, 41, 42, 57, 61.
5. **Section 10.3**, Problems 19, 23, 32, 42.
6. **Complex Notes** All the exercise in the notes.

To hand in

1. Show that $\displaystyle\sum_{k=0}^{\infty}\frac{\sin k}{2^k}$ converges. Evaluate it using the fact that $\sin k = \operatorname{Im}\left(e^{ik}\right)$.
2. Prove by inducton on $k \geq 0$ that $\dfrac{1}{(1-x)^{k+1}} = \displaystyle\sum_{n=0}^{\infty}\binom{n+k}{k}x^n$ when $|x|<1$, where $\dbinom{n}{k} = \dfrac{n!}{k!(n-k)!}$
3. Find all 4 (complex) roots of the equation $z^4+z^2+1=0$.
4. Let z and w be complex numbers.
   (a) Prove that $|z+w| \leq |z|+|w|$. This is the triange inequality. Draw a picture of a triangle with sides given by the inequality.
   (b) Prove that $|z+w|^2+|z-w|^2 = 2\left(|z|^2+|w|^2\right)$.
5. Compute the limit
$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^2\sin\left(x^2\right)}$$
using Taylor expansions.