# Math 135, Winter 2015, Homework 5

## For practice - do not hand in

1. Solve the following first order differential equations. Check you answers by plugging the solution back into the DE.
   (a) $y' = x - 4xy$
   (b) $y' = \csc x + y \cot x$
   (c) $x^2 y' + 2xy = 8x^3$
   (d) $y' = xe^{y-x^2}$, $y(0) = 0$.
   (e) $(xy + x)dx = (x^2y^2 + x^2 + y^2 + 1)dy$
2. The equation $y' + P(x)y = Q(x)y^n$ is called the Bernoulli equation. It becomes a linear equation after the change of variable $y^{-n+1} = z$. Solve the equation $y(6y^2 - x - 1)dx + 2xdy = 0$ using this idea.
3. From TP, page 726, Problems 3, 5, 7, 9

## To hand in

1. Let $I = [0, 1]$ and $Y_n(t) = t^n$. Show that the sequence $(Y_n)$ is not Cauchy by computing $||t^n - t^m||$.
2. Let $I = [-\pi, \pi]$, and consider the function $f_0 : I \to \mathbf{R}$ defined by $f_0(t) = e^t$. Let $f_n$, $n = 0, 1, 2, \ldots$ be the sequence of functions on $I$ defined inductively by the formula

$$f_{n+1}(t) = \cos(t) + \frac{1}{2}\sin(t) f_n(t)$$

   So $f_1(t) = \cos(t) + \frac{1}{2}\sin(t)e^t$, $f_2(t) = \cos(t) + \frac{1}{2}\sin(t)(\cos(t) + \frac{1}{2}e^t \sin(t))$, etc.

   Show that $f_n$ converges uniformly to a continuous function and find the limit $\lim_{n\to\infty} f_n$.