Math 135, Winter 2015, Homework 7

For practice - do not hand in

1. From TP, page 311, problems 12-23.
2. Prove Theorem 1 form lecture and deduce Example 2 as a corollary.
3. Show that $f(t) = e^{t^2}$ is not of exponential order.
4. Compute the Laplace transforms of the following functions:
   (a) $f_1(t) = te^{4t}\cos(-2t)$, $f_2(t) = \cos^2(t)$, $f_3(t) = \sqrt{t}e^t$.
   (b) $f(t) = \begin{cases} 4, & t < 2, \\ t+2, & 2 \le t \le 5. \\ e^{-t}, & t > 5. \end{cases}$
5. Compute the inverse Laplace transforms of the following functions:
   (a) $F_1(s) = \dfrac{1}{s^2+2s+10}$, $F_2(s) = \dfrac{3s}{s^2+4s+13}$, $F_3(s) = \dfrac{2s+7}{s^2+6s+9}$.
   (b) $F_1(s) = \dfrac{s^2-6}{s^3+4s^2+3s}$, $F_2(s) = \dfrac{16}{s(s^2+4)}$, $F_3(s) = \dfrac{6s-3}{s(s+1)^2}$.
   (c) $F(s) = \dfrac{(1-e^{-2s})(1-3e^{-2s})}{s^2}$

To hand in

1. (a) Use

$$\int \mathrm{Re}\left(e^{(a+bi)t}\right) dt = \mathrm{Re}\left(\int e^{(a+bi)t} dt\right)$$

   where $\mathrm{Re}(z)$ is the real part of a complex number z, to compute $\int e^{at}\cos bt dt$.
   (b) Find $\mathcal{L}\{\cos(bt)\}$.
   (c) Use Theorem 3 from lecture to find $\mathcal{L}\{\sin(bt)\}$.
   (d) Use Theorem 5 from lecture to find $\mathcal{L}\{t\sin(bt)\}$.
   (e) Use Theorem 1 from lecture to find $\mathcal{L}\{e^{at}\sin(bt)\}$.
   (f) Compute $\mathcal{L}\{te^{at}\sin(bt)\}$.
2. Solve the initial value problem $y'' + y = f(t)$, $y(0) = 0$ and $y'(0) = 0$ where

$$f(t) = \begin{cases} 4, & 0 \le t \le 2, \\ t+2, & t > 2. \end{cases}$$

3. Compute $\mathcal{L}\{h\}$ where

$$h(t) = \begin{cases} t, & 0 \le t < 1, \\ h(t-1), & t \ge 1, \end{cases}$$

   is the sawtooth wave function.