Math 135, Winter 2015, Homework 8

For practice - do not hand in

1. Page 546 of TP. For 3 and 4 do not use series methods. There are simpler methods. The $\log x$ in question 7 is $\ln x$. Do two of 5-7 with both methods of series solutions.

2. Solve $y'' + 3ty' + 3y = 0$ using series (the second method). Find the rule for ALL coefficients of the series like the last example from class.

3. Solve $(1 + t^2)y'' - 4ty' + 6y = 0$.

4. Find the formulas for the functions whose series expansions are given:

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)x^{2k}}{5^k}$.

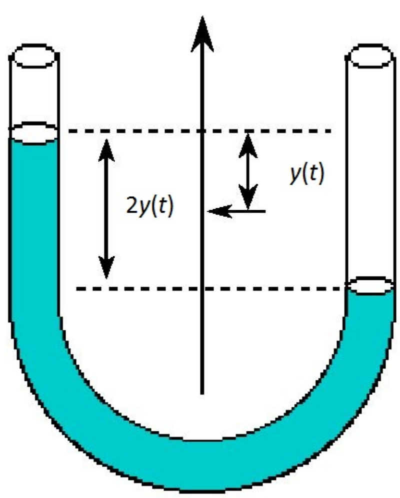
   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k+3)x^{2k+5}}{2^{2k}}$.

5. Find the series solution of $(t^2 + 4)y'' + 6ty' + 4y = 0$ using the second (undetermined coefficients) method. After you find the complete series (see the last example in class), write down the formula of the functions represented by the series (see question 4 above).

To hand in

1. Find the series solution of $(1 - 4t^2)y'' + 8y = 0$. After you find the complete series, write down the formula of the functions represented by the series. See question 5 above.

2. Consider a "U"-shaped tube filled with liquid Mercury as shown in the figure below. The radius of the tube is 1 centimeter (so its diameter is 2 centimeters). There are 500 grams of Mercury in the tube. Liquid Mercury has a mass density of 13.5 grams per cubic centimeter.



The mercury in the tube will oscillate with a certain period T, measured in seconds. Your task is to compute T by completing the following steps:

   (a) Let $y(t)$ be the height above its equilibrium position of the liquid surface at the left vertical segment of the tube. (At equilibrium, both surfaces are at the same height above sea level and $y = 0$. When $y(t) < 0$ the right surface is higher than the left surface.) The only force acting on the mass of fluid in the tube is due to gravity. Compute the total force on the fluid (vertical component only) and use that formula to find a linear, homogeneous, constant coefficient, second order differential equation for $y = y(t)$.

(b) Compute T by solving the differential equation you found in the previous part.

Answers to practice problems

1. In TP.

2.
$$y(t) = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-3)^k x^{2k}}{2^k k!}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-3)^k x^{2k+1}}{3 \cdot 5 \cdot \ldots \cdot (2k+1)}\right]$$

3.
$$y(t) = a_0(1 - 3t^2) + a_1\left(t - \frac{1}{3}t^3\right)$$

4. (a) $\dfrac{10x^2 - x^4}{(5 - x^2)^2}$.

   (b) $\dfrac{48x^5 + 4x^7}{(4 + x^2)^2}$.

5.
$$y(t) = a_0\left[1 + \sum_{k=1}^{\infty} \frac{(-1)^k (k+1) x^{2k}}{2^{2k}}\right] + a_1\left[x + \sum_{k=1}^{\infty} \frac{(-1)^k (2k+3) x^{2k+1}}{3 \cdot 2^{2k}}\right] = a_0\left[1 - \frac{x^4 + 8x^2}{(4 + x^2)^2}\right] + a_1\left[x - \frac{20x^3 + 3x^5}{3(4 + x^2)^2}\right]$$