Midterm 2 Practice Problems

Basics

1. Describe the set of points satisfying $x^2 - 4x + 2y^2 + 6y + z^2 - z = 5$.
2. Find a vector that has the same direction as $< -2, 3, 1 >$ but has length 3.
3. Determine if the vectors $< 1, 2, 4 >$ and $< -2, 4, -1 >$ are orthogonal.
4. Determine if the vectors $< 1, 0, 3 >$ and $< -2, 0, 6 >$ are parallel.
5. Sketch and describe the surface $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$.
6. Sketch and describe the curve $\mathbf{r}(t) =< \sin t, t, 2\cos t >$.
7. Find the unit tangent vector $\mathbf{T}$ for $\mathbf{r}(t) =< t, \sin(2t), \cos(2t) >$ when $t = 0$. Sketch the curve and the unit tangent vector at $t = 0$.
8. Find parametric equations for the tangent line to $\mathbf{r}(t) =< t^2 - t + 1, t, \sin(t) >$ at the point $(1, 0, 0)$.
9. Evaluate $\int_0^1 (16t^3\mathbf{i} - 9t^2\mathbf{j} + 25t^4\mathbf{k})dt$.
10. Find the unit tangent vector $\mathbf{T}$, the unit normal vector $\mathbf{N}$, the binormal vector $\mathbf{B}$ and the curvature κ for the curve $\mathbf{r}(t) =< 2t - 1, t^2, t^3 >$ at the point $(-1, 0, 0)$.
11. A particle's position is given by the vector function $\mathbf{r}(t) =< t, \sin(2t), \cos(2t) >$. Find the velocity and the acceleration of the particle at time t. Write the acceleration as a sum of two vectors, one tangent to the path, the other normal to the path.
12. Find the equation of the normal plane to $\mathbf{r}(t) =< t, 3, \cos(2t) >$ at $(0, 3, 1)$.
13. Find the length of the curve $\mathbf{r}(t) =< t, \sin(2t), \cos(2t) >$ from $t = 1$ to $t = \pi$.
14. Reparametrize the line $\mathbf{r}(t) =< t, 3t - 5, -2t + 3 >$ with respect to arclength measured from the point$(1, -2, 1)$.
15. Find the points of the polar curve $r = 1 + \sin\theta$ where the tangent is horizontal or vertical. Sketch the curve.
16. Determine the domain of the function $f(x, y) = \ln(1 - x^2 - y^2)$.
17. Find f_x, f_y, f_{xx}, f_{xy} and f_{yy} for the function $f(x, y) = x^2y^3 - \frac{1}{1-xy}$

Problems

1. Find an equation for the set of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$.
2. Find the equation of the plane through the points $(2, 1, 6)$, $(2, -1, 2)$ and $(6, 2, 0)$.
3. Describe the set of all points P whose distance from $A(-1, 5, 3)$ is twice the distance from $B(6, 2, -2)$
4. Write $< 2, 3 >$ as a sum of two vectors, one parallel to $< 1, 2 >$ and the other normal to $< 1, 2 >$.
5. Determine if $(1, 0, 1)$, $(2, 4, 6)$, $(3, -1, 2)$ and $(6, 2, 8)$ lie on the same plane.
6. Use vectors to find the distance from the point $(1, 2, 3)$ to the line through $(0, 2, 3)$ and $(1, 6, 4)$.
7. Find the distance between the skew lines $\mathbf{r}_1(t) =< 1+t, 1+6t, 2t >$ and $\mathbf{r}_2(s) =< 1+2s, 5+15s, -2+6s >$
8. Find the distance from the point $(0, 1, 2)$ to the plane $2x - 3y + z = 5$.
9. Find the angle between the planes $2x - 4y - z = 3$ and $5x + z = 2$.
10. Find parametric equations for the intersection of the planes $2x - 4y - z = 3$ and $5x + z = 2$.

11. Find a vector function that represents the curve of intersection of $z = 4x^2 + y^2$ and $y = x^2$.

12. At what point do the curves $\mathbf{r}_1(t) =< t, 1-t, 3+t^2 >$ and $\mathbf{r}_2(s) =< 3-s, s-2, s^2 >$ intersect? Find their angle of intersection.

13. Find the point where the curvature of the curve $\mathbf{r}(t) =< t^2 - 3t, t^2 + 2t >$ is maximum.

More Problems

1. Show that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

2. Find the angle between the diagonal of a cube and a diagonal of one of its faces.

3. Show that $\frac{d}{dt}(\mathbf{r} \times \mathbf{r}') = \mathbf{r} \times \mathbf{r}''$

4. If a curve has the property that the position vector $\mathbf{r}(t)$ is always perpendicular to the tangent vector $\mathbf{r}'(t)$, the curve lies on a sphere with center at the origin.

5. Show that

   (a) $\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}$.

   (b) $\frac{d\mathbf{N}}{ds} = -\kappa\mathbf{T} + \tau\mathbf{B}$. ($\tau$ is the torsion of the curve. It is given by $\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}$)