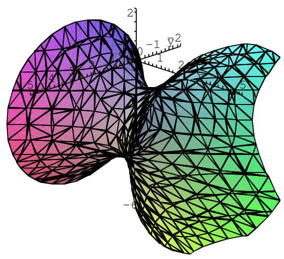
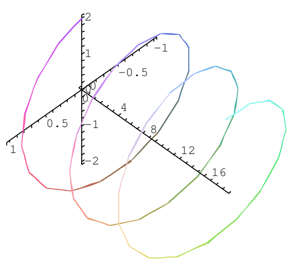
Answers to Practice Problems

Basics

1. Ellipsoid with center at $(2, -3/2, 1/2)$.
2. $< -\frac{3\sqrt{14}}{7}, \frac{9\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} >$
3. No.
4. No.
5. Hyberboloid of one sheet.

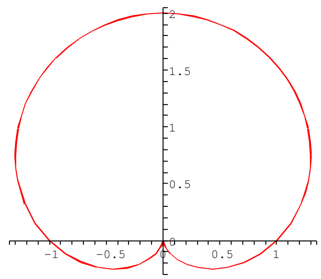


6. This is like a helix but it wraps around an elliptic cylinder.



7. $\mathbf{T}(0) =< \frac{\sqrt{5}}{5}, 2\frac{\sqrt{5}}{5}, 0 >$.
8. $x = 1 - t$, $y = t$, $z = t$.
9. $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$.
10. $\mathbf{T}(0) =< 1, 0, 0 >$, $\mathbf{N}(0) =< 0, 1, 0 >$, $\mathbf{B}(0) =< 0, 0, 1 >$ and $\kappa = 1/2$.
11. $\mathbf{v}(t) =< 1, 2\cos(2t), -2\sin(2t) >$ and $\mathbf{a}(t) =< 0, -4\sin(2t), -4\cos(2t) >$. The acceleration is normal to the path.
12. $x = 0$.
13. $\sqrt{5}(\pi - 1)$.
14. $\mathbf{r}(s) =< 1 + \frac{\sqrt{14}}{14}s, 3(1 + \frac{\sqrt{14}}{14}s) - 5, -2(1 + \frac{\sqrt{14}}{14}s) + 3 >$.

15. The tangent is horizontal at $(2, \pi/2)$, $(1/2, 11\pi/6)$ and $(1/2, 7\pi/6)$. The tangent is vertical at $(3/2, \pi/6)$ and $(3/2, 5\pi/6)$.



16. Inside the unit circle.
17. $f_x = 2xy^3 - \frac{y}{(1-xy)^2}$, $f_y = 3x^2y^2 - \frac{x}{(1-xy)^2}$, $f_{xx} = 2y^3 - \frac{2y^2}{(1-xy)^3}$, $f_{xy} = 6xy^2 - \frac{2xy}{(1-xy)^3}$ and $f_{yy} = 6x^2y - \frac{2x^2}{(1-xy)^3}$.

Problems

1. $14x - 6y - 2z = 9$.
2. $2(x-2) - 2(y+1) + (z-2) = 0$.
3. Sphere.
4. $< 2, 3 >=< \frac{8}{5}, \frac{16}{5} > + < \frac{2}{5}, -\frac{1}{5} >$.
5. Yes.
6. $\frac{17}{18}\sqrt{2}$.
7. 2.
8. $\frac{3\sqrt{14}}{7}$.
9. About 1.18 radians.
10. $x = \frac{2-t}{5}$, $y = \frac{-11-7t}{20}$ and $z = t$.
11. $\mathbf{r}_1(t) =< t, t^2, 4t^2 + t^4 >$
12. $(1, 0, 4)$ about 0.96 radians.
13. $(4, 24)$.

More Problems

1. Show that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
2. About 0.62 radians.
3. Show that $\frac{d}{dt}(\mathbf{r} \times \mathbf{r}') = \mathbf{r} \times \mathbf{r}''$
4. If a curve has the property that the position vector $\mathbf{r}(t)$ is always perpendicular to the tangent vector $\mathbf{r}'(t)$, the curve lies on a sphere with center at the origin.
5. Show that
   (a) $\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}$.
   (b) $\frac{d\mathbf{N}}{ds} = -\kappa\mathbf{T} + \tau\mathbf{B}$. ($\tau$ is the torsion of the curve. It is given by $\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}$)