Hw 6

Read chapter 15-18, 19 of the textbook.
You need to know:

- The division theorem in ch 15 (no proofs)
- The definition of greatest common divisor
- The definition of congruence mod m

Do the following problems from your textbook:

- p 239: 19.2
- p. 225:2,3

. Do the following additional problems.

1. Is the statement $\forall a \in Z \quad \forall n \in Z \; n \text{ div } a \Leftrightarrow n \text{ div } a^2$ True or false ? Justify your answer.
2. Prove that $(a, b) = d \Rightarrow (\frac{a}{d}, \frac{b}{d}) = 1$
3. Prove that if $(a, b) = 1$ then $\forall x \in Z \; ab \text{ div } x \Leftrightarrow a \text{ div } x \wedge b \text{ div } x$.
4. Prove that if $(a, b) > 1$ then it is not true that $\forall x \in Z \; ab \text{ div } x \Leftrightarrow a \text{ div } x \wedge b \text{ div } x$.
5. Prove that if $a = bq + r$ with $0 \leq r < b$ then $(a, b) = (b, r)$. (This is the main idea in the euclidean algorithm)