I. (a) $\nabla\times\vec{F}=\langle z, x, x\rangle$.

(b) The equation with the given vertices is $z=p-x-y$, so $d\vec{S}=\pm\langle 1,1,1\rangle\ dx\ dy$. We want the outward pointing normal to be consistent with the direction of C : we use the plus sign. The shadow, D, on the x,y plane is the triangle bounded by the line $x+y=p$ and the positive axes. Thus

$$\int_C \vec{F}\cdot d\vec{S}=\iint_S \nabla\times\vec{F}\cdot d\vec{S}=\iint_D (z+2\text{x})dx\ dy=\int_0^p\int_0^{p-x}(p+x-y)\ dy\ dx=\frac{1}{2}p^3$$

(c) $p=2^{1/3}$

II. (a) The circle has parameterization $\vec{R}=\langle\sqrt{2\text{m}}\cos t, \sqrt{2\text{m}}\sin t, m\rangle$, which is consistent with the orientation of S. The surface integral becomes, by Stoke's Theorem, the line integral $\int_C -zy\ dx+xz\ dy+yz^2\ dz.$ Because $z = m$ is constant, this integral simplifies greatly:

$$\int_C -zy\ dx+xz\ dy+yz^2\ dz=m\int_C -y\ dx+x\ dy=m(\sqrt{2\text{m}})^2\int_0^{2\pi}(\cos^2 t+\sin^2 t)dt=4\text{m}^2\pi .$$

(b) $m=\dfrac{1}{2\sqrt{\pi}}$.

III. (a) The upper surface, call it S_1, has normal $\hat{n}=(1/a)\langle x, y, z\rangle$, so

$\vec{F}\cdot\hat{n}=(1/a)(x^2z^2+y^2z^2+z^4+4\text{z})=(1/a)[z^2(x^2+y^2+z^2)+4\text{z}]=az^2+4\text{z}/a$.

So we must evaluate the surface area integral: $\iint_{S_1}(az^2+4\text{z}/a)dS.$ We have a "function surface": $z=\sqrt{a^2-x^2-t^2}$, $x^2+y^2\le a^2$. A standard formula then applies:

$$dS=\sqrt{(\partial z/\partial x)^2+(\partial z/\partial y)^2+1}\,dx\,dy=\frac{a\ dx\ dy}{\sqrt{a^2-x^2-z^2}}=(a/z)\,dx\,dy.$$

Consequently

$$\iint_{S_1}(az^2+4\text{z}/a)dS=\iint_D (a/z)(az^2+4\text{z}/a)dxdy=\iint_D (a^2\sqrt{a^2-x^2-y^2}+4)dxdy ,$$

where D is the disc of radius a centered at (0,0). Now use polar coordinates to complete the evaluation. The result is $\iint_{S_1}\vec{F}\cdot n\,dS=(2/3)\pi a^5+4\pi a^2$. For S_2, we take $\hat{n}=-\hat{k}.$ Then $\vec{F}\cdot\hat{n}=-4$, and $dS=dxdy.$ So $\iint_{S_2}\vec{F}\cdot n\,dS=-4\pi a^2$. Adding the two integrals together yields :

$$\iint_S \vec{F}\cdot\hat{n}\,dS=\iint_{S_1}\vec{F}\cdot\hat{n}\,dS+\iint_{S_2}\vec{F}\cdot\hat{n}\,dS=(2/3)\pi a^5.$$

(b) The divergence is $\nabla\cdot\vec{F}=5\text{z}^2$. Switching to shperical coords, the divergence integral becomes

$$\iiint_E \nabla\cdot\vec{F}\,dV=\int_0^{2\pi}\int_0^{\pi/2}\int_0^a 5(\rho\cos\phi)^2\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta=(2/3)\pi a^5.$$

IV. The correct answers for #1-#8 are BADBBCBC, in that order.